Probability Density of a Partially Coherent Gaussian Laser Beam in Environmental Turbulence

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Abstract

The present study aims to find how power distribution using various base probability density functions and window size affects the statistical properties of a laser after passing through a turbulent environment. The collected data varied in environment as well as beam polarization. A partially coherent beam was propagated through a controlled turbulent environment in which temperature and air velocities were varied. The results are provided as graphs.

Kernel Method

- The Experiment aims to generate a probability density function (PDF) for the power distribution of a laser beam. A PDF allows one to predict the likelihood of an event occurring.
- The likelihood of it occurring between two a and b according to function f is:

$$P(a < X < b) = \int_a^b f(x)dx \quad \text{for all } a < b.$$

- The estimate of an event occurring is known as density estimate.
- The oldest, widest used, and most basic is the histogram, which can be described by the equation:

$$f(x) = \frac{1}{n} \times \frac{(\text{no. of } X_i \text{ in the same bin as } x)}{(\text{width of bin containing } x)}$$

- The Histogram provides a base to analyze the data.
- Many shortcomings caused by discontinuity including interpolating data density between the bins.
- I use the Kernel Method for data estimation as developed by Silverman.
- The individual PDF the K function must fulfill the following equation [1]:

$$\int_{-\infty}^{\infty} K(x)dx = 1.$$

- The Kernel estimator generates the PDF by summing the K function. The PDF is defined by the equation [1]:

$$\text{PDF} = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right)$$

where h is the window width and Xi denotes the i-th element of the data array.

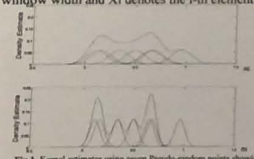

Fig.1. Kernel estimates using seven Pseudo-random points showing individual kernels and effect of the window width on each individual kernel shape. Window widths: (a) 0.1; (b) 0.05

- An h value that is too small provides a function which generates false statistical properties.
- An h value that is too large creates too large of a spread and eliminates nearly all data facets.

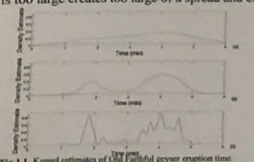

Fig.1.1. Kernel estimates of Old Faithful geyser eruption time. Window widths: (a) 0.8; (b) 0.25; (c) 0.05 [1]

- The Density estimation function K that I use are based on three common PDFs.

The normal Gaussian distribution curve where μ=0 and σ=1.

$$K(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The Cauchy distribution where x0=0 and γ=1.

$$K(x) = \frac{1}{\pi\gamma\left[1 + \left(\frac{x-x_0}{\gamma}\right)^2\right]}$$

And the Laplace distribution where μ=0 and b=1.

$$K(x) = \frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right)$$

Probability Density Function Method

The PDF were generated using a MATLAB function based on a function provided by Professor Reza Malek-Madani. The Function has three inputs: the data, the window width (*h*), and the x value.

PDF Algorithm:

For loop (repeat for every value in the data array)

Compute the term according to the Kernel method

Compute the individual probability density function at each point using one of the base PDFs

Add the new PDF to the previous PDF to generate the sum via the Kernel method

Function for a Normal Distribution

```
function PDF=KernelMethodnormal(DATA,x,h)
nn=length(DATA); %number of data points
sum=0;
for i=1:length(DATA) %use a for loop to sum the K function
term=((x-DATA(i))/h).^2; %input term according to Kernel method
sum=sum+1/nn/h*1/sqrt(2*pi)*exp(-term/2); %Normal distribution function
end
PDF=sum; %set the sum of the K functions equal to the PDF
end
```

Experimental Set-up

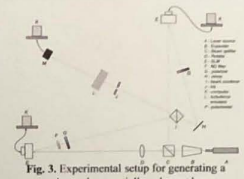

Fig. 3. Experimental setup for generating a pseudo-random partially coherent beam.

The experiment included splitting Gaussian beam into two independent branches, and then simultaneously modulating the beam out of phase to produce a partially coherent beam using computer controlled SLMs. The beam was polarized and either recombined with the perpendicular polarization or left as a single polarized beam and then passed through a controlled turbulent environment. The beam's power was then recorded by a polarimeter to record the power at the target. Each experiment was carried out over 1024 independent readings at a sampling time of 30 micro seconds.

Results

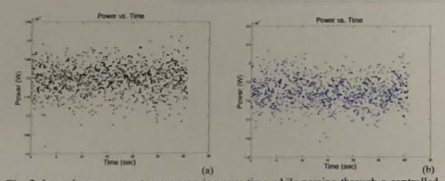

Fig. 3. Laser beam power at sensor varying over time while passing through a controlled turbulent environment. The horizontal heat guns outputting at 400 °F and at the high speed and the vertical heat guns outputting at 200 °F and at the base speed. (a)vertically polarized laser and (b) horizontally polarized laser.

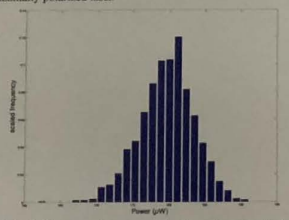

Fig.4. Scaled frequency histogram of the data collected in Fig 3.

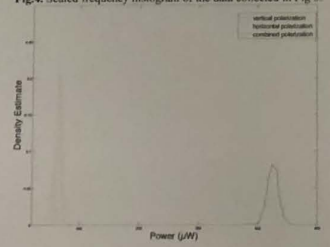

Fig.7. Comparison of the probability density function for vertical, horizontal, and combined polarization laser beam passing through a controlled turbulent environment. The horizontal heat guns outputting at 200 °F and at the high speed and the vertical heat guns outputting at 800 °F and at the base speed. Probability density function using window width 0.8 and using a normal Gaussian distribution as the K function. Density estimate scaled by 10.

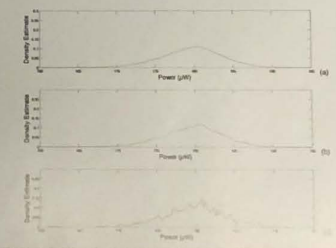

Fig.5. Probability density function from data collected in Fig. 3. PDF uses a normal Gaussian distribution as the K function. Window widths: (a) 1; (b) 0.5; (c) 0.1. Density estimate scaled by 10.

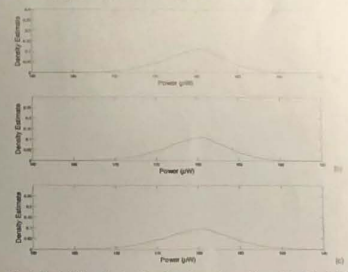

Fig.6. Probability density function from data collected in Fig.3 The PDF uses a window width of 0.8. K functions: (a) normal Gaussian distribution; (b) Laplace distribution; (c) Cauchy distribution. Density estimate scaled by 10.

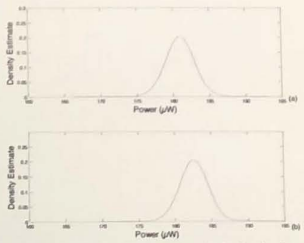

Fig.8. Comparison of the probability density function for a vertically polarization laser beam passing through a controlled turbulent environment at two different temperatures. The horizontal heat guns outputting at 200 °F and at the high speed and the vertical heat guns outputting at (a) 400 °F (b) 800 °F and at the base speed. Probability density function using window width 0.8 and using a normal Gaussian distribution as the K function. Density estimate scaled by 10.

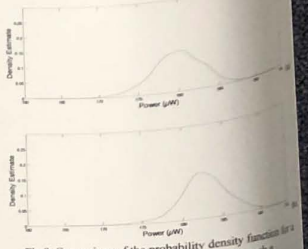

Fig.9. Comparison of the probability density function for a vertically polarization laser beam passing through a controlled turbulent environment at two speed orientations. (a) The horizontal heat guns outputting at 200 °F and at the high speed and the vertical heat guns outputting at 200 °F and at the base speed. (b) The horizontal heat guns outputting at 200 °F and at the base speed and the vertical heat guns outputting at 200 °F and at the high speed. Probability density function using window width 0.8 and using a normal Gaussian distribution as the K function. Density estimate scaled by 10.

Conclusion

- Through visually interpolating the shape changes in the PDF in Fig. 5 the optimal window width appears to be 0.8
- Fig. 6 demonstrates the effect of changing the K function on PDF. The Laplace and Cauchy distribution functions provide a PDF with a longer tail.
- Fig. 7 demonstrates that a vertically polarized beam produces a narrower PDF than the horizontal or combined beam at termination when traveling through a turbulent environment.
- Fig. 8 demonstrates an increase in temperature lessens the effect of power loss.
- Fig. 9 indicates a vertically polarized partially coherent beam passing through an environment with greater turbulence parallel to the polarization generates a smother PDF as opposed to an increase in perpendicular turbulence.
- These PDFs provide immense benefit for communications as it allows for a better understanding of how turbulence will manipulate the power distribution as the information transmitted over a turbulent environment.

Data transmitted by a laser is inherently secure as it cannot be intercepted without a detectible disruption in the transmission [

Reference

[1] B. W. Silvennan, "Density Estimation For Statistics And Data Analysis," Vol. [illegible] Chapman & Hall/CRC Monographs on Statistics & Applied Probability.

[illegible] (M.S. in Applied Science)--Naval Postgraduate School, 1977. Issued [illegible] Postgraduate School Technical [illegible] NPS61-77-001) in conjunction with William M. Tolles.

[illegible] Zhao; Lyke, S.D.; Roggemann, M.C., "Adaptive Optical Communications [illegible] Turbulent Atmospheric [illegible] Communications, 2008. ICC '08. IEEE International Conference on [illegible] pp.5432,5436, 19-23 May 2008.

[illegible] Zhao, Bei Wang, Li Zhou, Longyan Gong, Wenwen Cheng, [illegible] Zheng, Baoyu Zheng, "Turbulence Mitigation Scheme for Optical Communications using Orbital Angular Momentum Multiplexing Based on Channel Coding and Wavefront Correction," arXiv:1401.7558 [physics.optics].

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