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Propagation of J0-Bessel-correlated beams in weak atmospheric turbulence

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Motivation:

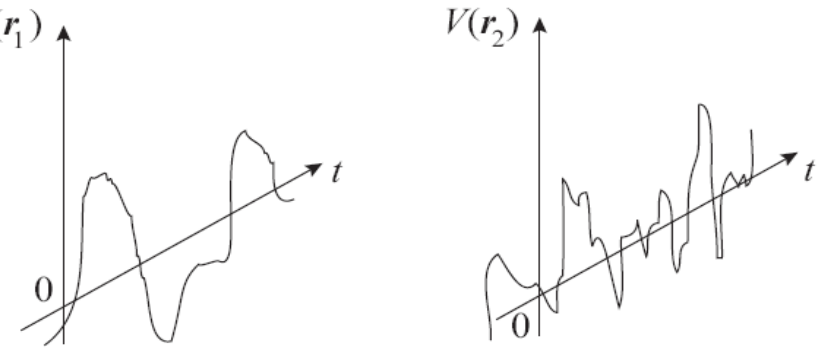
Experimentally explore the idea of correlation-shape diversity of sources generating random beams for efficient atmospheric propagation.

Outline

1. Correlation diversity of random beams
2. Known correlation shapes
3. J_0-Bessel-Gaussian correlation family
4. Optical setup for generation of random beams
5. Formation of the J0-Bessel-Gaussian beams
6. Atmospheric propagation of the beams
7. Summary

1. Correlation shape diversity of random beams

Fluctuating optical fields:



Correlation function: $W(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega) = f[S(\boldsymbol{\rho}_1''; \omega)] g[S(\boldsymbol{\rho}_2''; \omega)] \mu(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega)$

Schell-model:

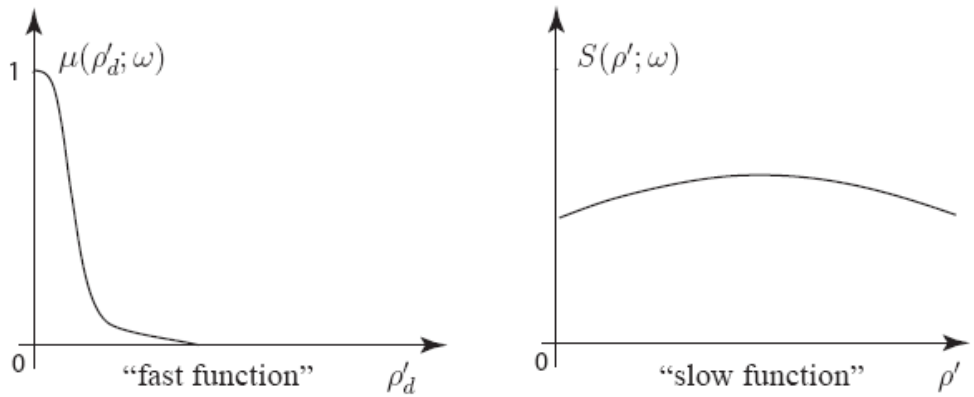
$f(x) = g(x) = \sqrt{x} \qquad \boldsymbol{\rho}_1'' = \boldsymbol{\rho}_1', \boldsymbol{\rho}_2'' = \boldsymbol{\rho}_2'$

$$W(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega) = \sqrt{S(\boldsymbol{\rho}_1'; \omega)} \sqrt{S(\boldsymbol{\rho}_2'; \omega)} \mu(\boldsymbol{\rho}_1' - \boldsymbol{\rho}_2'; \omega)$$

Quasi-homogeneous model:

$f(x) = x, g(x) = 1 \qquad \boldsymbol{\rho}_1'' = (\boldsymbol{\rho}_1' + \boldsymbol{\rho}_2') / 2$

$$W(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \omega) = S\left(\frac{\boldsymbol{\rho}_1' + \boldsymbol{\rho}_2'}{2}; \omega\right) \mu(\boldsymbol{\rho}_1' - \boldsymbol{\rho}_2'; \omega)$$



Far field intensity: $S^{(\infty)}(r\mathbf{u}; \omega) \propto \tilde{\mu}(k\mathbf{u}_\perp; \omega)$

2. Known correlation shapes

1. Gaussian:

$$\mu^{(G)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)=\exp\left[-\frac{(\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2')^2}{\delta^2}\right]$$

2. J0-Bessel:

$$\mu^{(J)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)=J_0(\beta\,|\,\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2'\,|)$$

3. J0-Bessel-Gaussian:

$$\mu^{(BG)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)=J_0\left(\frac{\beta}{\delta}\,|\,\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2'\,|\right)\exp\left[-\frac{(\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2')^2}{\delta^2}\right]$$

4. Laguerre-Gaussian:

$$\mu^{(BG)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)=L_n\left(\frac{\beta}{2\delta}\,|\,\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2'\,|\right)\exp\left[-\frac{(\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2')^2}{2\delta^2}\right]$$

5. Cosine-Gaussian:

$$\mu^{(CG)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)=\cos\left(\frac{n\sqrt{2\pi}}{\delta}\,|\,\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2'\,|\right)\exp\left[-\frac{(\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2')^2}{2\delta^2}\right]$$

6. Multi-Gaussian circular :

$$\mu^{(MG)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)=\sum_{m=1}^{M}\binom{M}{m}\frac{(-1)^{m-1}}{m}\exp\left[-\frac{(\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2')^2}{2m\delta^2}\right]$$

rectangular:

$$\mu^{(RG)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)=\sum_{m=1}^{M}\binom{M}{m}\frac{(-1)^{m-1}}{\sqrt{m}}\exp\left[-\frac{(x_1'-x_2')^2}{2m\delta_x^2}\right]$$

$$\times\sum_{m=1}^{M}\binom{M}{m}\frac{(-1)^{m-1}}{\sqrt{m}}\exp\left[-\frac{(y_1'-y_2')^2}{2m\delta_y^2}\right]$$

7. Non-uniform

$$\mu^{(NU)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)=\exp\left[-\frac{\{(\boldsymbol{\rho}_1'-\boldsymbol{\gamma})^2-(\boldsymbol{\rho}_2'-\boldsymbol{\gamma})^2\}^2}{\delta^4}\right]$$

8. I0-Bessel

$$\mu^{(IB)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)=\frac{I_n(2\rho_1'\rho_2'/\delta^2)}{\sqrt{I_n(2\rho_1'/\delta^2)}\sqrt{I_n(2\rho_2'/\delta^2)}}\exp\left[-in(\phi_1'-\phi_2')\right]$$

3. J0-Bessel-Gaussian correlation family

Source degree of coherence :

$$\mu^{(BG)}(\boldsymbol{\rho}_1',\boldsymbol{\rho}_2';\omega)=J_0\left(\frac{\beta}{\delta}\,|\,\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2'\,|\right)\exp\left[-\frac{(\boldsymbol{\rho}_1'-\boldsymbol{\rho}_2')^2}{\delta^2}\right]$$

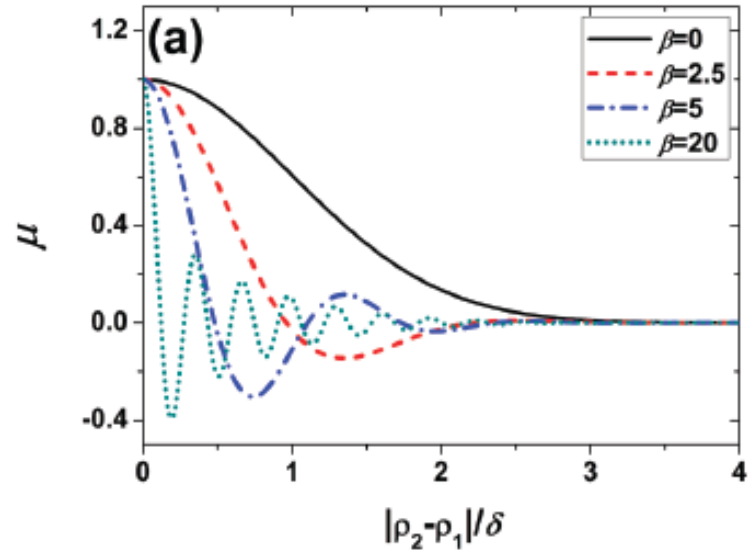
Source intensity distribution:

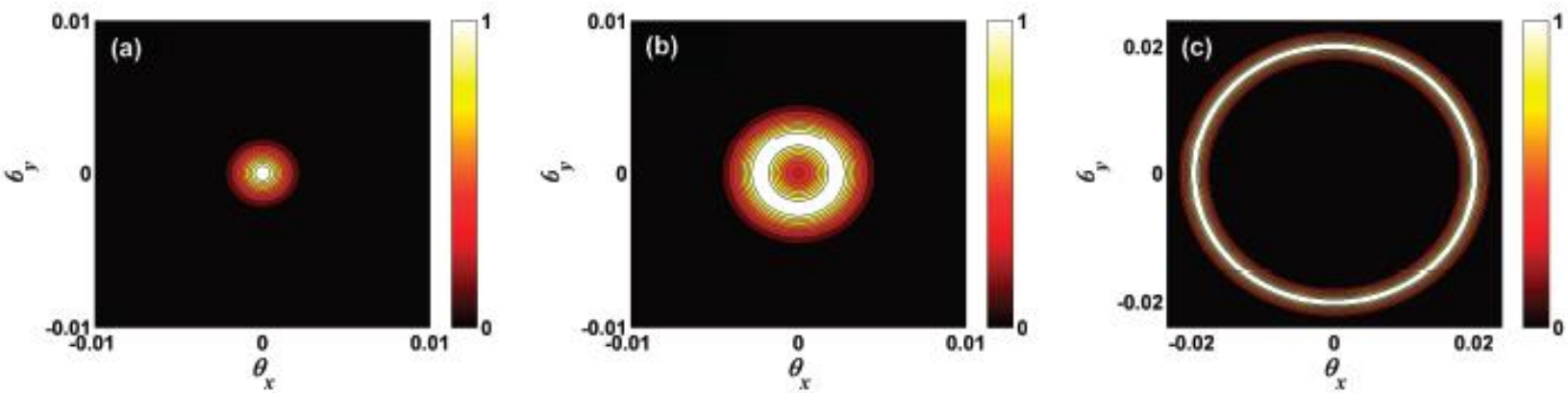
$$S^{(G)}(\rho';\omega)=\exp\left[-\frac{\rho'^2}{\sigma^2}\right]$$

Theoretically predicted far field:

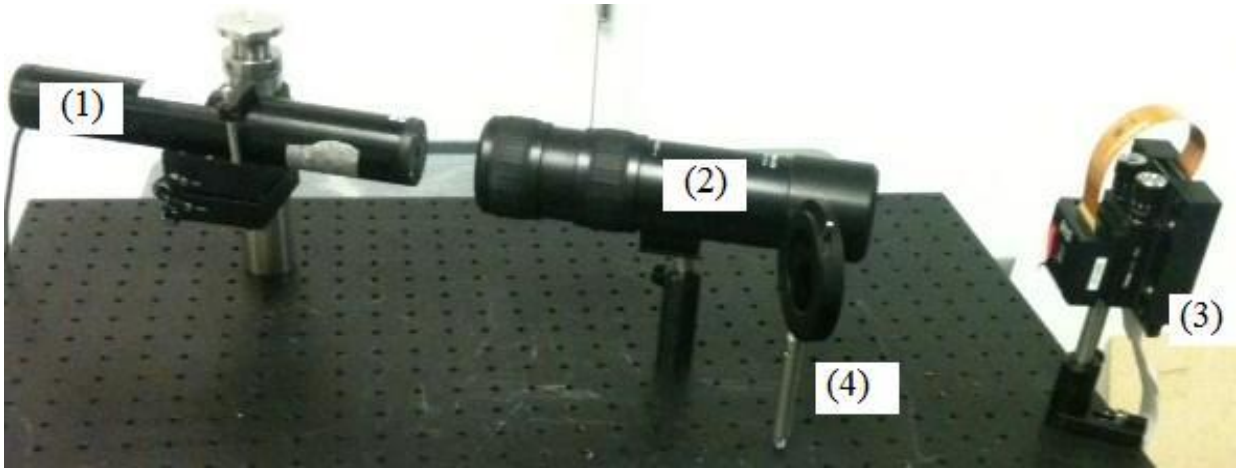
$$S_{(\infty)}^{(BG)}(r\mathbf{s};\omega)=\frac{k^2\sigma^2}{2br^2}s_z I_0\left(\frac{\beta}{2b\delta}u_\perp\right)\exp\left[-\frac{\beta^2}{4b\delta^2}-\frac{k^2}{4b}u_\perp^2\right]$$

$$\theta_{\max}=\frac{\sqrt{16\beta^2-56b\beta^2\delta^2+b^2\delta^2}+4\beta^2-b\delta^2}{8k\beta\delta}\qquad b=1/8\sigma^2+1/2\delta^2$$





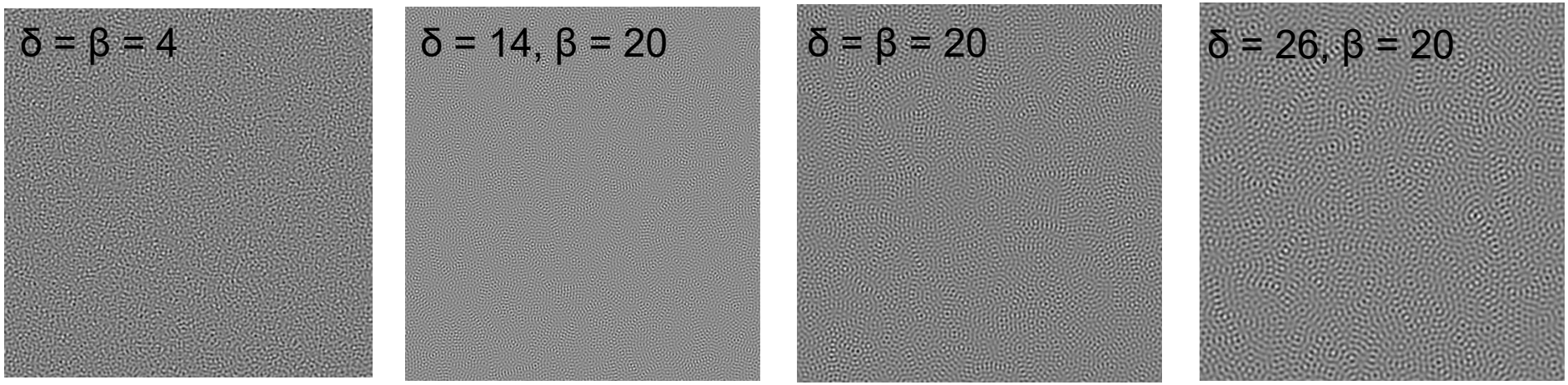
4. Optical setup for generation of random beams



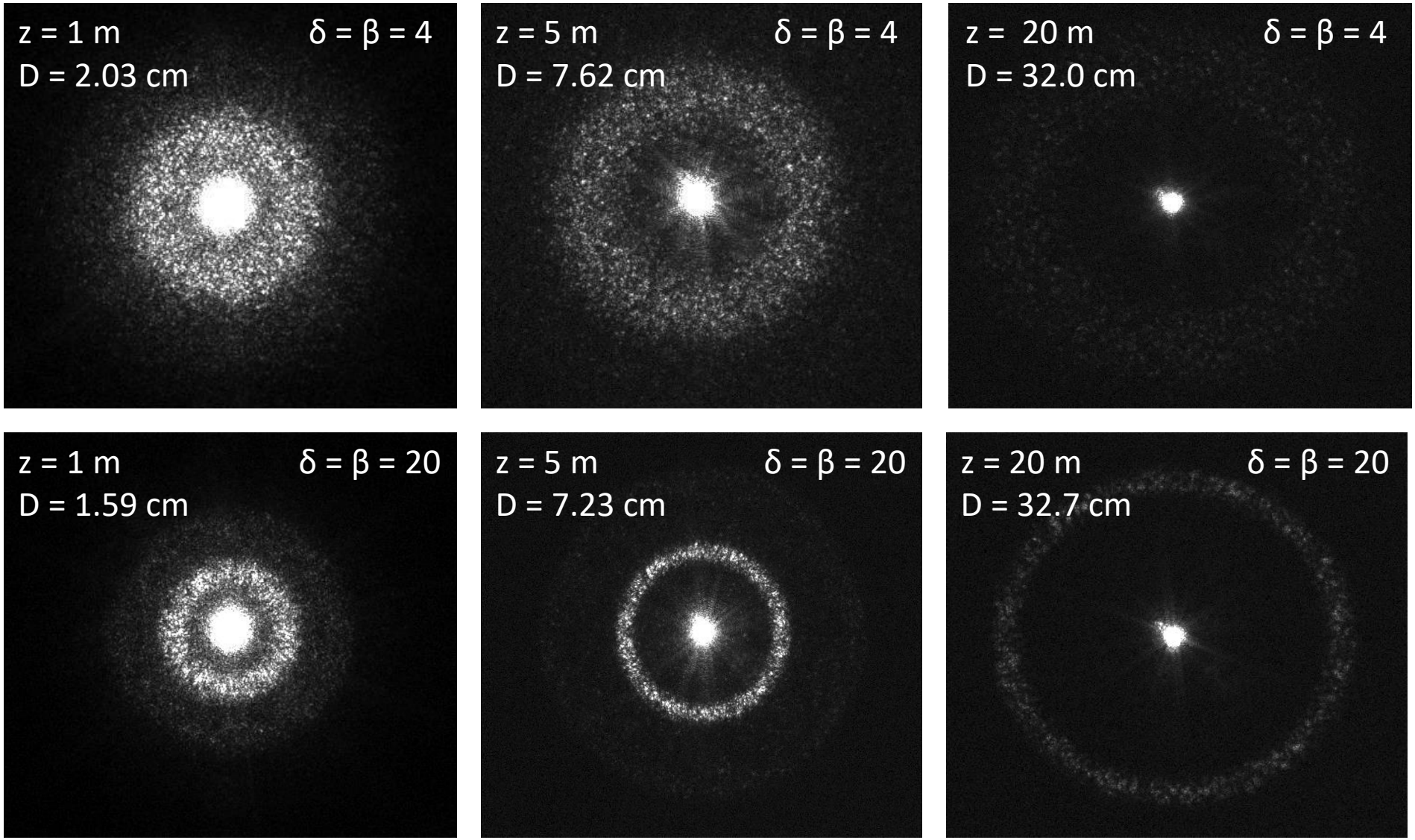
(1) laser source;
(2) beam expander;
(3) SLM;
(4) iris.

Distances between the devices: (1)-(2) 5 cm; (2)-(3) 10 cm; (3)-(4) 15 cm.
Expander length 25 cm.

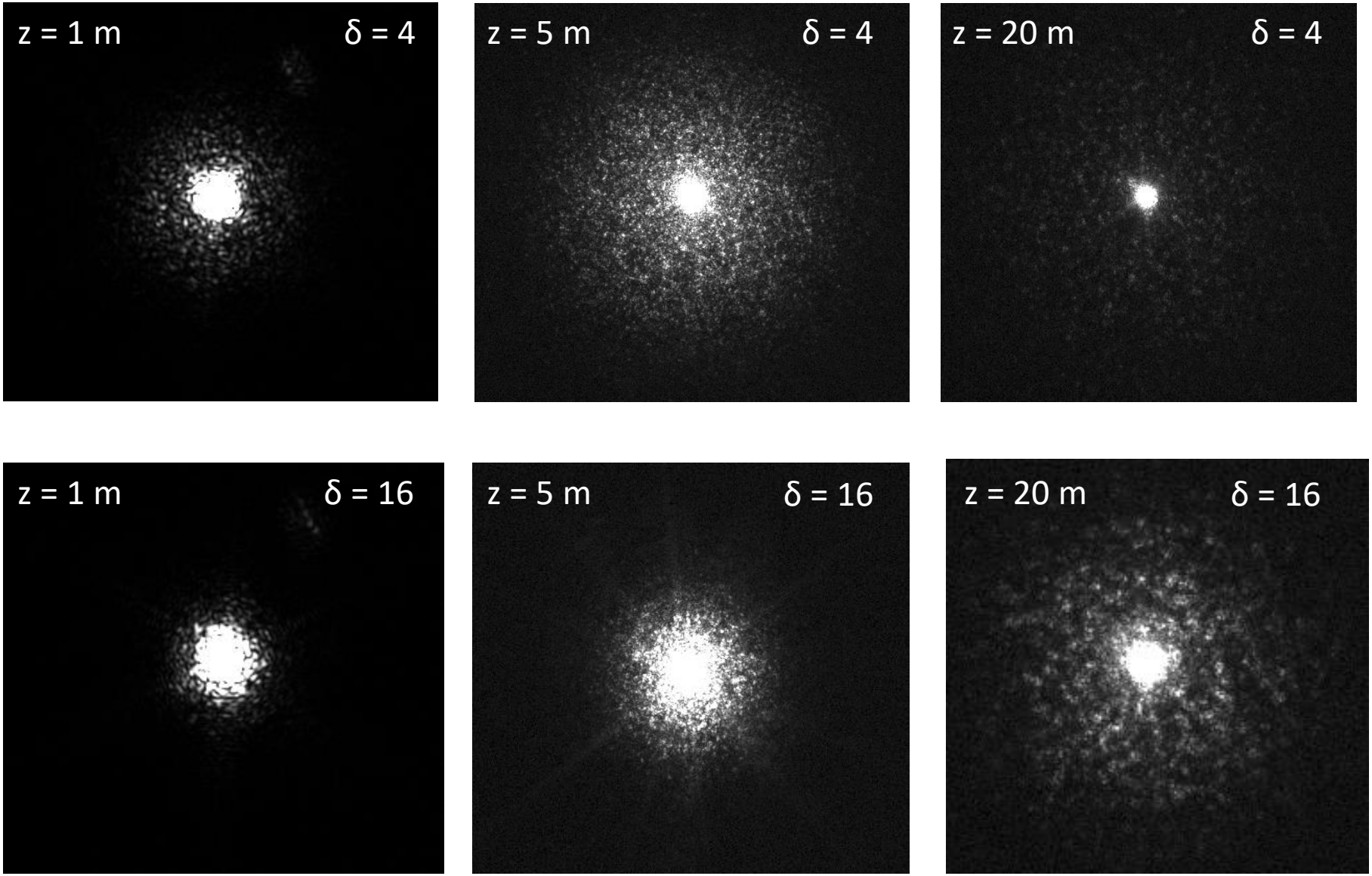
$$\delta, \beta[mm] = \frac{7.68\,mm}{512} \times \sqrt{\delta, \beta[pixels]}$$



5. Formation of J_0-Bessel-correlated beams

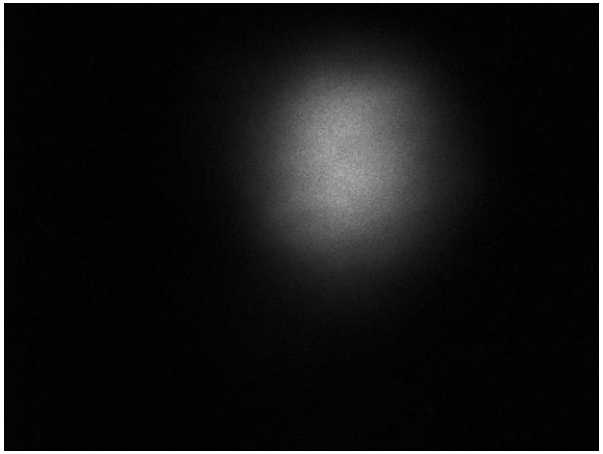


Formation of reference (Gaussian correlated) beams



6. Atmospheric propagation of differently correlated beams

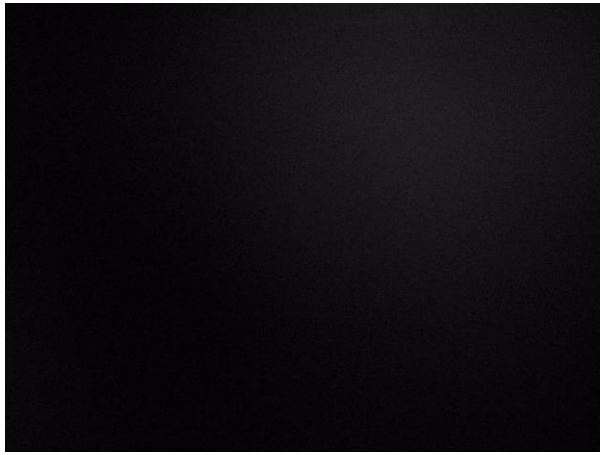
On central spot



J_0-Bessel-Gaussian correlation

$\delta = \beta = 20$

In the ring



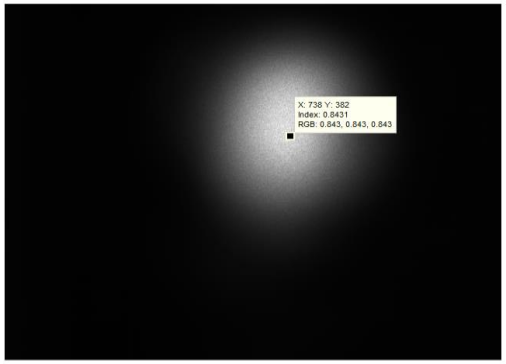
Gaussian correlation

$\delta = 16$

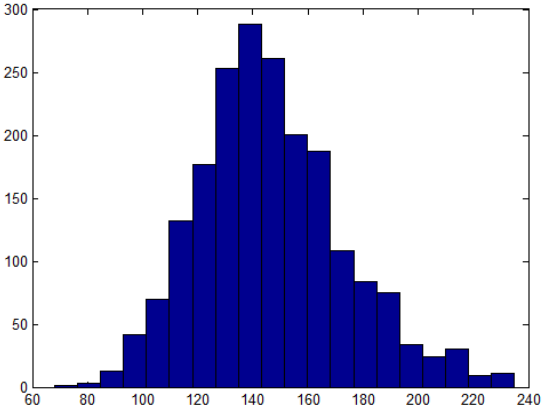
Off-center

Data analysis

On spot

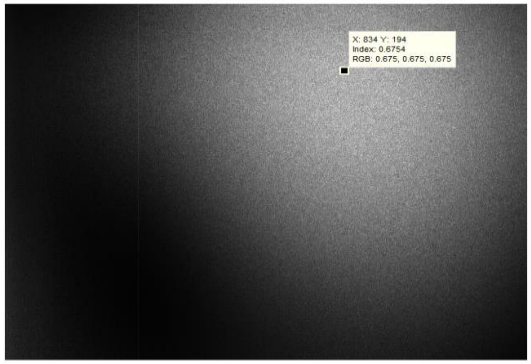


Avg. int. = 146.5
SI = 0.0327

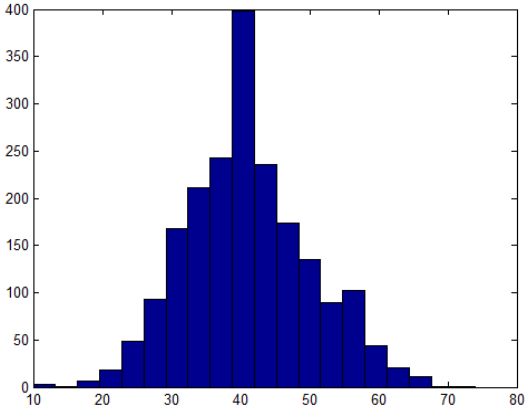


J0-Bessel-Gaussian

δ = β = 20

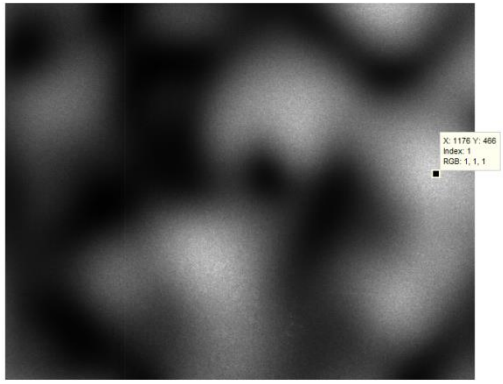


Avg. int. = 32.1
SI = 0.0487

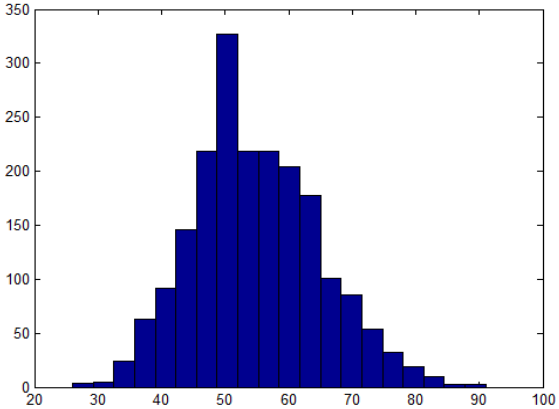


Gaussian

δ = 16



Avg. int. = 54.9
SI = 0.0335



7. Summary

• J_0-Bessel-Gaussian correlated beams are generated with the help of the reflective SLM. Predicted by the theory ring-shaped far fields occur at distances of several meters for typical source parameters.

• On propagation in the atmosphere the ring shape (for generic 20-20 beam) is preserved at distances of at least 350 meters, with a radius of about 4 meters.

• The diffraction size of the J_0-Bessel-Gaussian correlated beams excides (as measured in the ring) that of the similar Gaussian-correlated beam (taken as a 1/e point), hence its average intensity is lower but scintillations are higher.

• Thus within the family of J_0-Bessel-Gaussian correlated beams Gaussian correlated beams show the best performance (in terms of divergence and scintillation), while J_0-Bessel correlated beams - the worst.