Probability density function of fluctuating intensity of laser beam propagating in marine atmospheric turbulence

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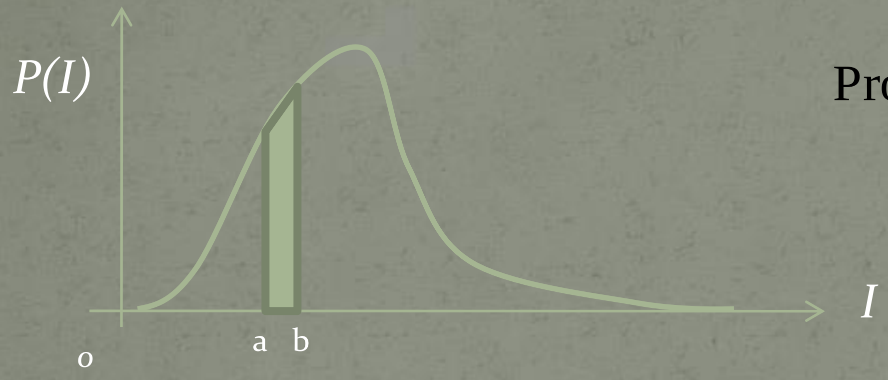
Goal:
Probability Density Function reconstruction of the fluctuating beam intensity in atmospheric marine environment

- **Experiment:**
  - Instrumentation set up in the field at the United States Naval Academy
  - Measure intensity statistics of the beam in its transverse cross-section during daylight.
- **Theory:**
  - Calculations of statistical moments of intensity
  - Applying several PDF reconstruction models
  - Comparison among the models and data sets

Probability Density Function

- PDF of fluctuating intensity P(I) shows with which chance the beam's intensity attains a certain level.



$$\text{Probabilit y}(a < I < b) = \int_a^b P(I)dI$$

$$\mathrm{I}^{(l)} = \int_0^\infty P(I)I^l dI$$

- Determination of the PDF from the moments is an academically noble problem: (famous Hausdorff moment problem)
- The knowledge of the PDF is crucial for solving inverse problems finding statistics of a medium
- The tails of the PDF affect the fade statistics of a signal encoded in a beam (BER errors in a communication channel)

THEORY

Probability Distribution Function Reconstruction Methods

- Gamma distribution modulated by series of generalized Laguerre polynomials proposed by Barakat
  - Medium and source independent
  - Uses first n moments of detected intensity
  - Valid in the presence of scatterers
- Gamma- Gamma distribution based on the work of Nakagami et. al. and presented by Andrews and Philips
  - Medium and source dependent
  - Uses 2 first moments
  - Valid only in clear air atmosphere

Gamma-Laguerre Model

Richard Barakat Vol. 16, No. 9/September 1999/J. Opt. Soc. Am. A 2269

First-order intensity and log-intensity probability density functions of light scattered by the turbulent atmosphere in terms of lower-order moments

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Processing procedure

1. Calculation of statistical moments of fluctuating intensity from data

$$\left\langle h^{(l)}(x,y)\right\rangle = \sum_{k=1}^{k_{\max}} \frac{\left[h_k(x,y)\right]^l}{k_{\max}}$$

h	Fluctuating intensity
k	Index of realization
$k_{\max}$	Total number of realizations
(x,y)	Coordinates of the pixel

2. Fitting the moments into the Probability Density Function

Note: $\left\langle h^{(l)}\right\rangle = \int W(h) h^l \, dh$

The proposed intensity PDF is[23–25]

$$W(h) = W_g(h) \sum_{n=0}^{\infty} W_n L_n^{(\beta-1)}\left(\frac{\beta h}{\mu}\right), \tag{2.1}$$

where h is the scattered intensity ($0 \leq h < \infty$). Here $W_g(h)$ is the gamma PDF

$$W_g(h) = \frac{1}{\Gamma(\beta)}\left(\frac{\beta}{\mu}\right)^{\beta} h^{\beta-1} \exp(-\beta h/\mu), \tag{2.2}$$

where

$$\mu = \langle h \rangle$$

$$\beta = \frac{\langle h \rangle^2}{\langle h^2 \rangle - \langle h \rangle^2}. \tag{2.3}$$

The generalized Laguerre polynomials are

$$L_n^{(\beta-1)}(x) = \sum_{l=0}^{n} \binom{n+\beta-1}{n-l} \frac{(-x)^l}{l!}. \tag{2.4}$$

They are orthogonal with respect to $W_g(h)$. Using the orthogonality condition, we can show that the W_n expansion coefficients are given by

$$W_n = n!\Gamma(\beta) \sum_{l=0}^{n} \frac{(-\beta/\mu)^l \langle h^l \rangle}{l!(n-l)!\Gamma(\beta+l)}, \tag{2.5}$$

thereby expressing W_n directly in terms of the first n moments of h. Note that the W_n are linear functions of the moments. Furthermore,

$$W_0 = 1, \qquad W_1 = W_2 = 0, \tag{2.6}$$

so that

$$W(h) = W_g(h)\left[1 + \sum_{n=3}^{\infty} W_n L_n^{(\beta-1)}\left(\frac{\beta h}{\mu}\right)\right]. \tag{2.7}$$

Table 1. Values of the Measured Intensity Moments $\langle I^l \rangle^a$

Measured Intensity Moments	Value of Measured Moments
0	1
1	1.00
2	1.47
3	3.04
4	8.24
5	27.4

$^a\beta = 2.14$.

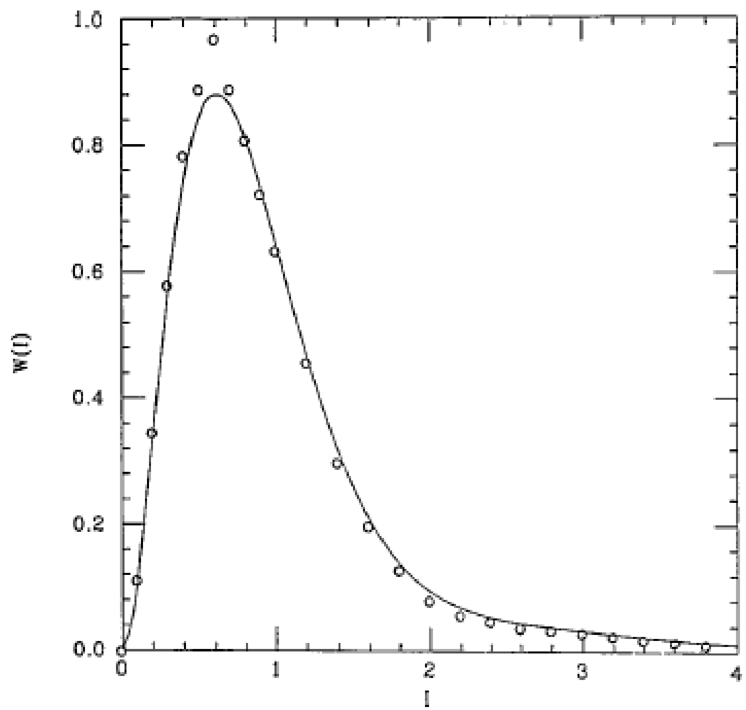


Fig. 1. Circles, experimental values of $W(I)$; curve, $W(I)$ as determined from the first five measured intensity moments.

Gamma-Gamma Model

Andrews and Phillips

$$P(I) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} I^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta I})$$

$\Gamma(x)$ - Gamma-function

$K_m(x)$ - Modified Bessel function of the second kind.

$$\alpha = \frac{1}{\exp(\sigma_{\ln x}^2) - 1} \qquad \beta = \frac{1}{\exp(\sigma_{\ln y}^2) - 1}$$

$\sigma_{\ln x}^2$ and $\sigma_{\ln y}^2$ are normalized variances of the fluctuating intensity due to large and small turbulent inhomogeneities, respectively.

For the Gaussian beam model and the Kolmogorov power spectrum model:

$$\sigma^2_{\ln x} = \frac{0.49\sigma_B^2}{[1+0.56(1+\theta)\sigma_B^{12/5}]^{7/6}} \qquad \sigma^2_{\ln y} = \frac{0.51\sigma_B^2}{[1+0.69\sigma_B^{12/5}]^{5/6}}$$

where σ_B^2 is the normalized variance of fluctuating intensity in the center of the beam:

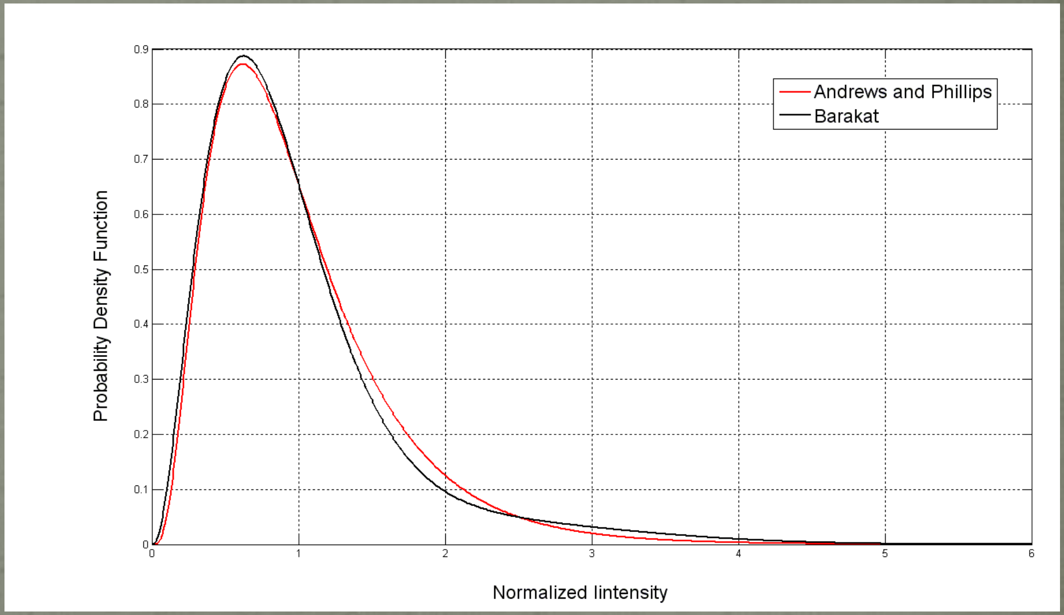
$$\sigma_B^2 = \frac{<I^2> - <I>^2}{<I>^2}$$

$$\theta = \frac{1}{1+\left(\frac{2L}{kW_0^2}\right)^2}$$

for collimated beams, with L being propagation distance from the source to the receiver, k is wave number and W_o is the initial beam radius (after the expander).

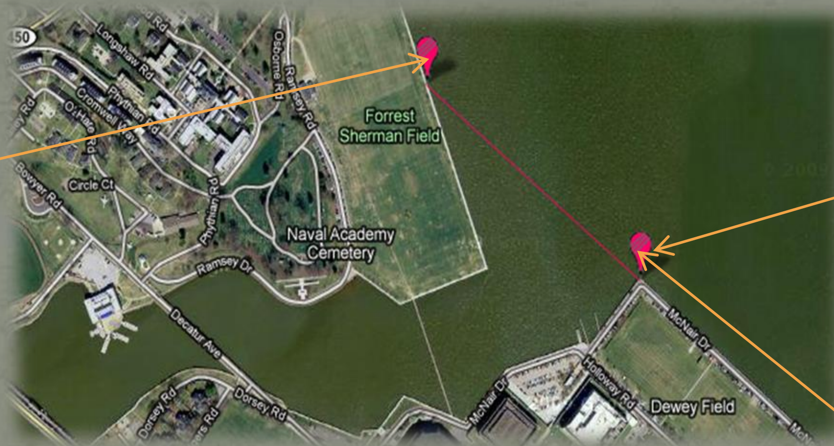
Comparison between Gamma-Laguerre and Gamma-Gamma Models

- Moments values provided by Barakat for both models
- Kolmogorov spectrum used for Gamma-Gamma model
- Source to target distance 400 m used in Gamma-Gamma model



EXPERIMENT

College Creek Test



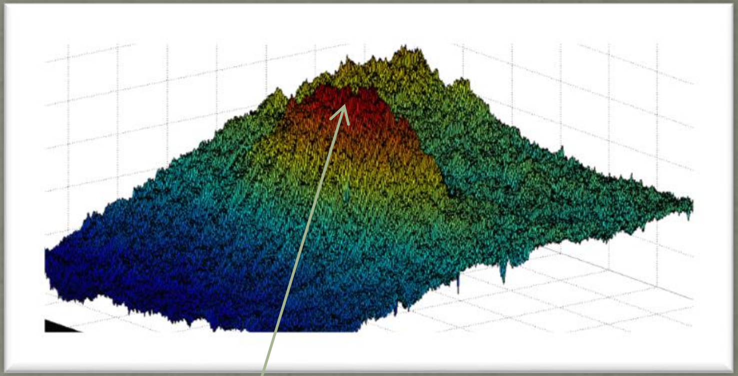
College Creek site with 400 m long laser link. Experiment 18 June 2010. Source is low power (4 mW) red He-Ne laser with an expander, creating 1 cm wide Gaussian beam. Target is a white board. Light intensities amplitude range of 255 was measured. Sequencing 30 frames per second three minutes of data collected. Pixel size was effectively measuring 0.3 mm^2 . Spacial coherence radius for our Gaussian beam is on the order of 1 cm.

Weather conditions at Hospital Point on June18, 2010							
Time	Temp	Humidity	Sea Level Pressure	Wind Dir	Wind Speed	Gust Speed	Conditions:
9:54 AM	73.0 °F	57%	30.13 in	North	none	none	Clear
10:54 AM	75.0 °F	53%	30.13 in	none	none	none	Clear

Data processing

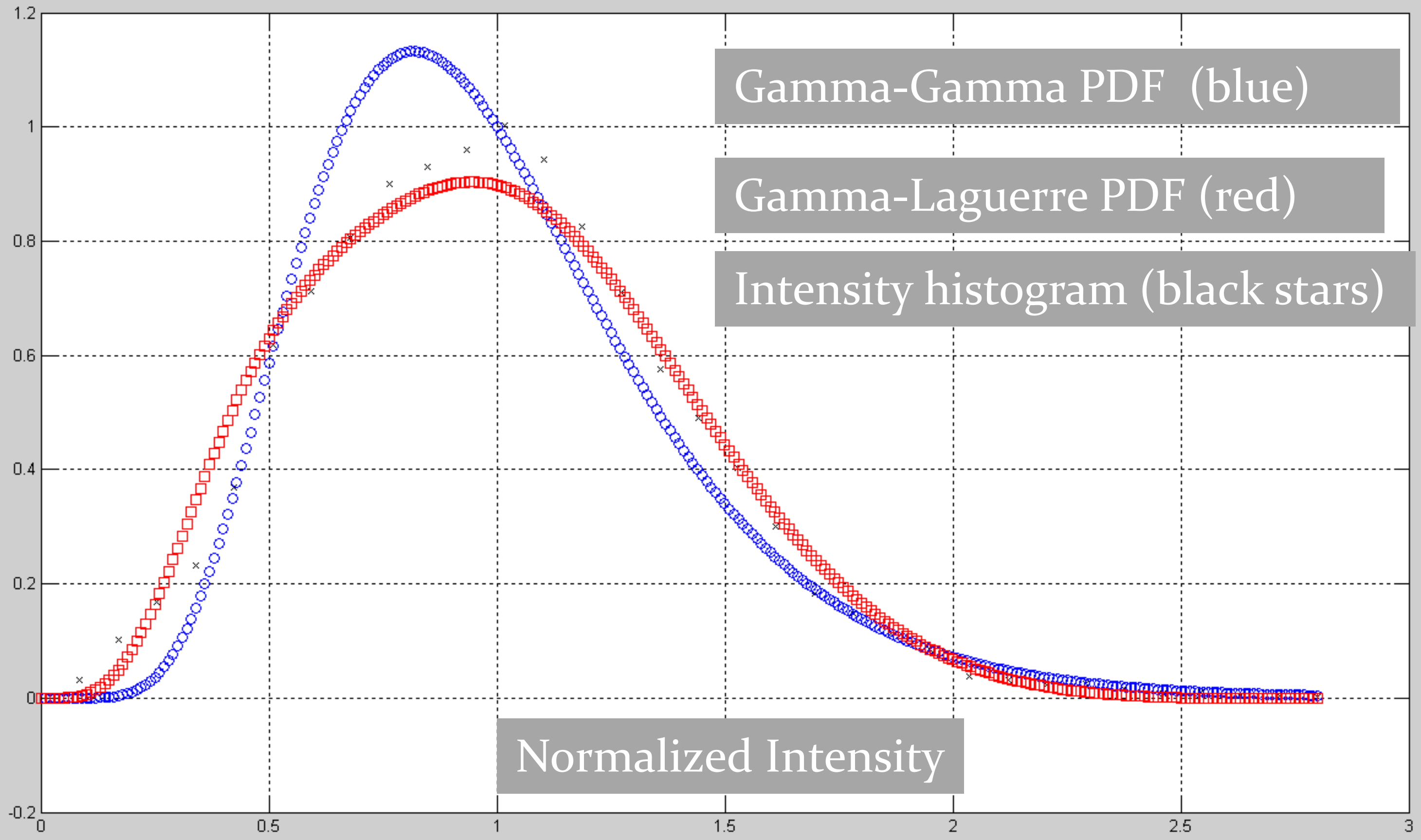
- All of the realizations were added to create cumulative intensity plot, P_{max}.
- From P_{max} location with maximum intensity was determined, $(x_{m,} y_m)$.
- Intensity vector , I, used for calculating PDF was formed by selecting intensity at the location $(x_{m,} y_m)$ for each realization.

- Cumulative intensity plot



Maximum intensity used for determination of $(x_{m,} y_m)$.

Propagation above the water

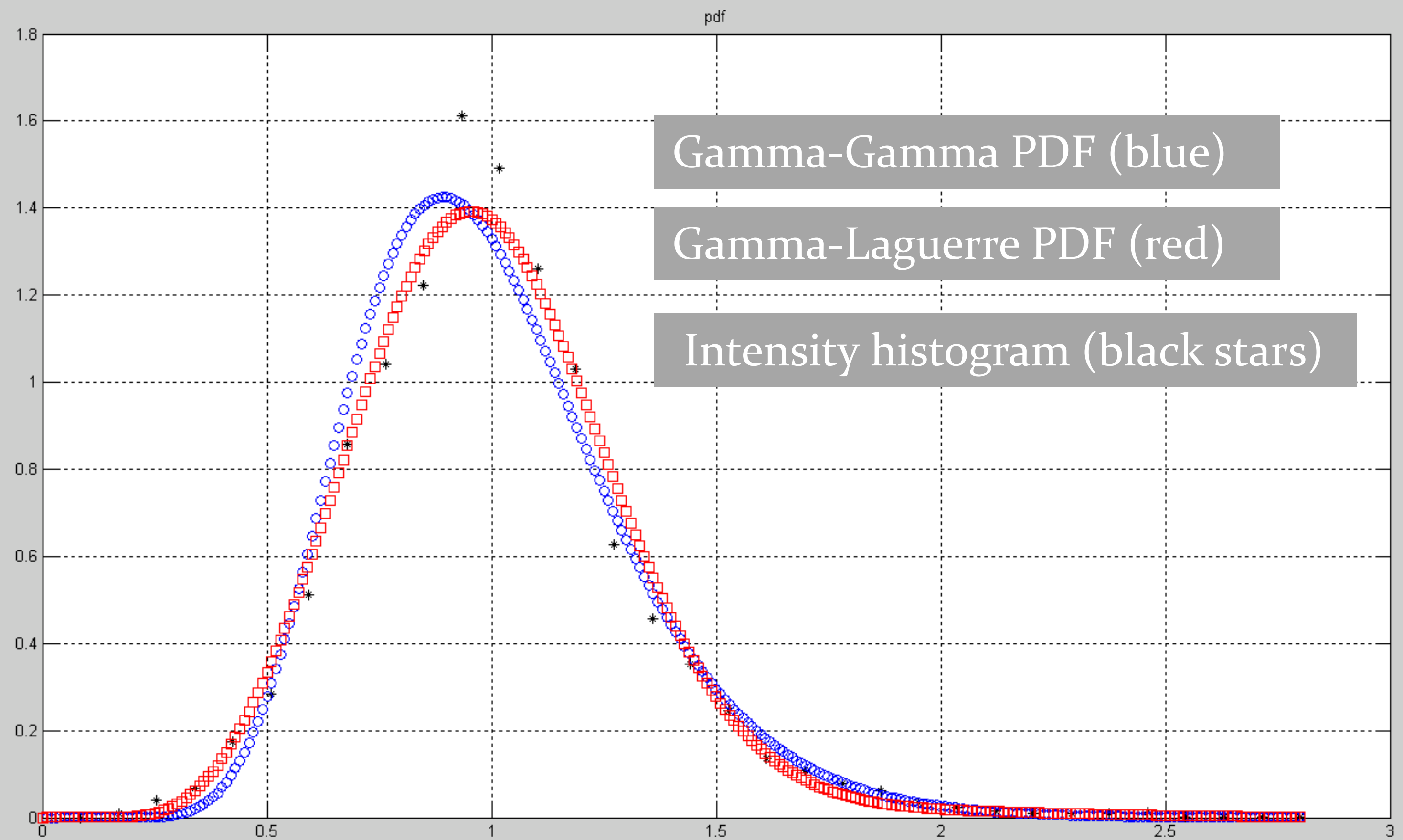


Forest Sherman Field Test

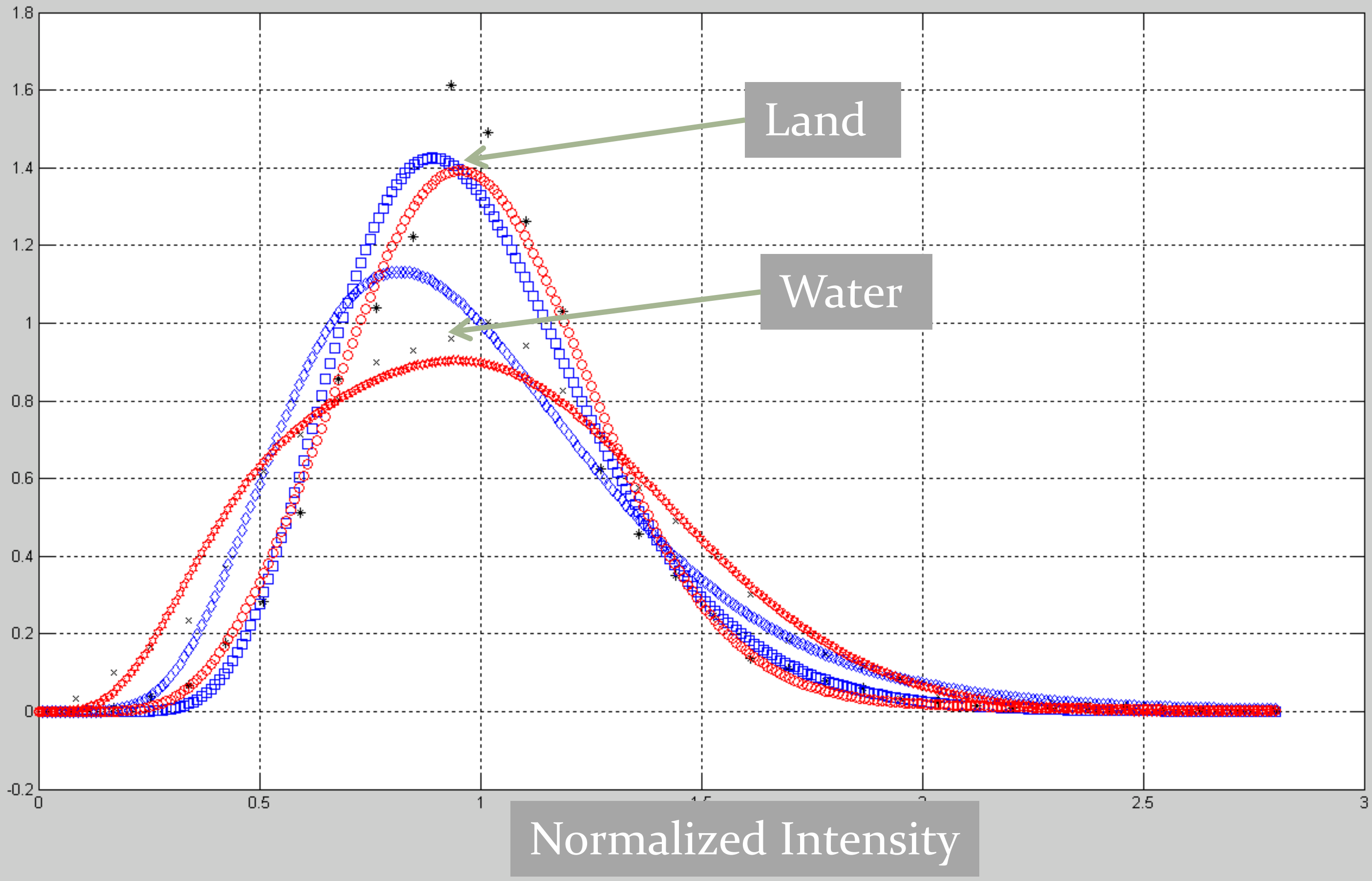
Forest Sherman Field site with 400 m long laser link. Experiment 15 June 2010. Source is low power (4 mW) red He-Ne laser with an expander, creating 1 cm wide Gaussian beam. Target is a white board.

Weather conditions at Hospital Point on June15, 2010							
Time	Temp	Humidity	Sea Level Pressure	Wind Dir	Wind Speed	Gust Speed	Conditions:
11:54 AM	79.0 °F	62%	30.08 in	NNE	8.1 mph	none	Partly Cloudy
12:54 PM	78.1 °F	62%	30.08 in	NNE	6.9 mph	none	Clear
1:54 PM	77.0 °F	66%	30.09 in	North	6.9 mph	none	Mostly Cloudy

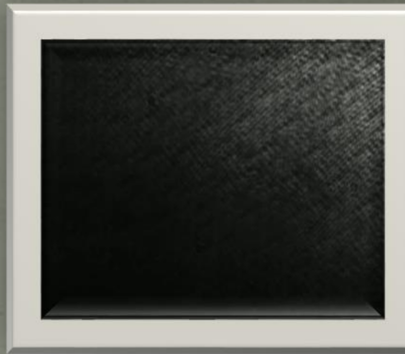
Propagation above the land

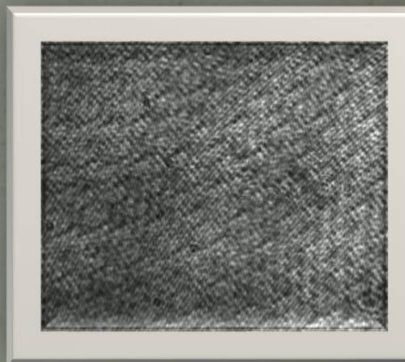


Propagation above the water and above the land



College Creek/Forest Sherman Field Test

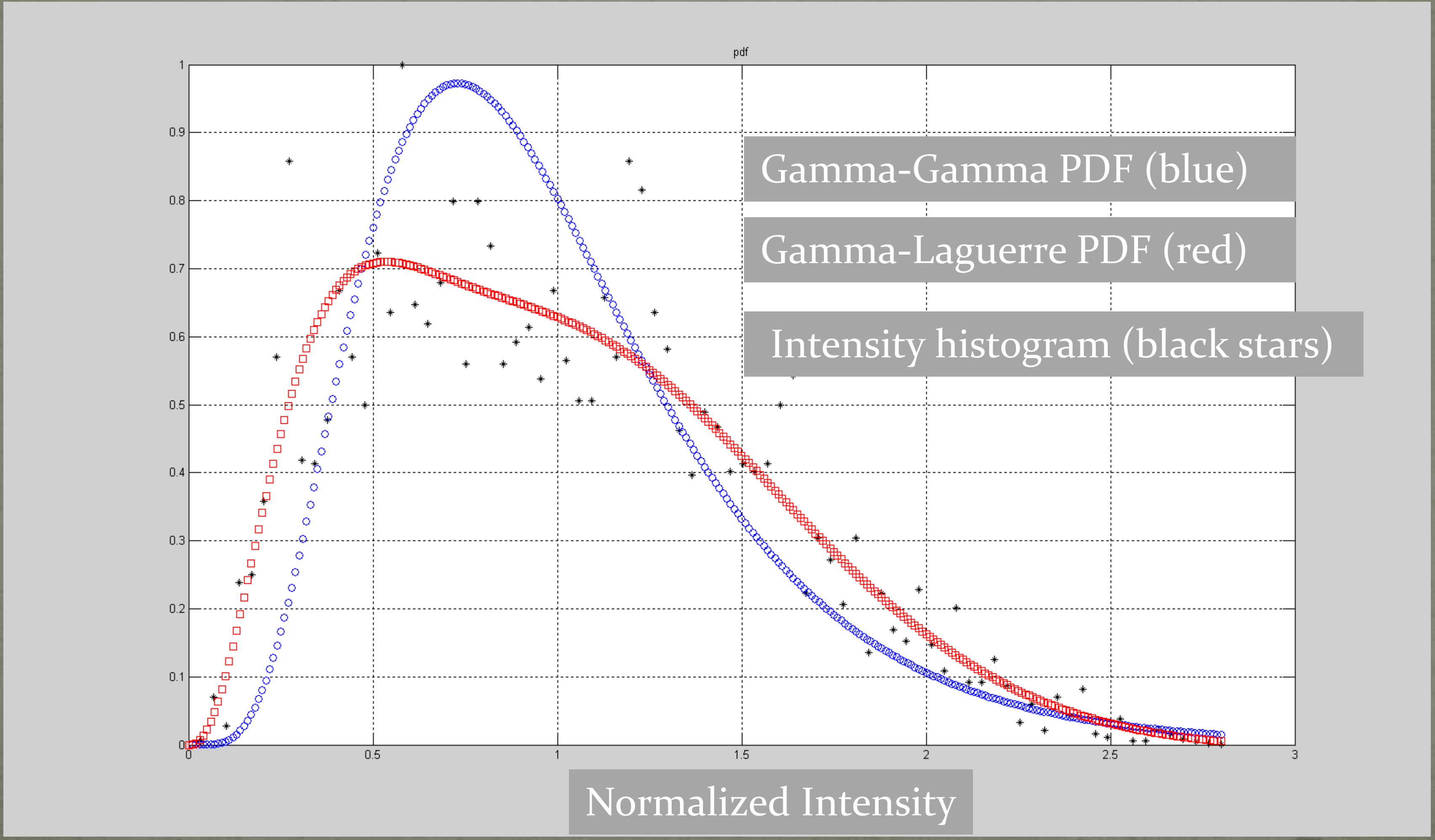




College Creek/Forest Sherman field site with 500m long laser link. Experiments on November 14 2010.
Source is low power (4 mW) red He-Ne laser with an expander, creating 1 cm wide Gaussian beam. Target is ccd sensor with red notch filter. The sensing area is 7.6 mm (horizontal) × 6.2 mm (vertical) with pixel size of 4.65 μm.

Weather conditions at Hospital Point on November 14, 2010							
Time	Temp	Humidity	Sea Level Pressure	Wind Dir	Wind Speed	Gust Speed	Conditions:
1:54 PM	55.9 °F	62%	30.04 in	ESE	8.1 mph	none	Clear
2:54 PM	55.9 °F	62%	30.03 in	SE	6.9 mph	none	Clear

Propagation on the boundary between the land and water (direct sensor measurements)



SUMMARY

1. We measure Gaussian laser beam propagation over the land in the proximity of the water and above the water.

2. Based on two different methods, Gamma-Laguerre, and Gamma-Gamma we reconstruct from collected data the single-point Probability Density Function (PDF) of the fluctuating intensity of a laser beam propagating through the marine type atmospheric turbulence.

3. We present two ways of measuring light intensity at the target, namely (a) pictures of reflection of the beam off a white board and (b) capturing light intensity directly using ccd sensor. The later method leads to a more refined PDF.

4. We present comparison of models with data histogram and find good agreement. In particular, Gamma-Laguerre model emphasizes the tails agreeing better with data histogram. This can be due to prevailed water particle scattering and absorption above the water column which suppress optical intensity fluctuations.

5. Our results will find uses for any applications involving radiation transfer through marine-type atmospheric turbulence.

- Fluctuating intensity of a laser beam propagating through ground and marine atmospheric channels was measured under weak atmospheric conditions. The data was fitted to the Barakat's model for the probability density function (PDF) which uses Gamma distribution for accounting for the first two statistical moments and generalized Laguerre polynomials for accounting for moments of orders higher than two. Comparison of the above-ground and the above-water PDFs is made and the dependence of the shape of the PDF of the intensity on the radial position within the propagating beam is revealed.