Probability Density Function Analysis for Optical Turbulence with Applications to Underwater Communications Systems

[a]Marius P. Bernotas, [a]Charles Nelson
[a]United States Naval Academy, 121 Buchanan Road, Annapolis, MD 21412

Abstract

The Weibull and Exponentiated Weibull probability density functions have been examined for the free space regime using heuristically derived shape and scale parameters. This paper extends current literature to the underwater channel and explores use of experimentally derived parameters. Data gathered in a short range underwater channel emulator was analyzed using a nonlinear curve fitting methodology to optimize the scale and shape parameters of the PDFs. This method provides insight into the scaled effects of underwater optical turbulence on a long range link, and may yield a general set of equations for determining the PDF for an underwater optical link.

Keywords: Underwater optical turbulence, optical communications, turbulence emulator, probability density functions, non-linear curve fitting, bit error rate, communications performance, beam propagation

1. Introduction

Transmission options in the realm of underwater wireless communications are extremely limited. Due to heavy absorption by water, electromagnetic communications in the radio frequency range are not viable, and acoustic communication methods, while effective, are limited in bandwidth to the kbps to low Mbps range [1]. Current demand for high bandwidth underwater communication systems is growing, and acoustic methods of transmitting data do not appear to be suited to shoulder the load.

The utility and effectiveness of laser based underwater optical wireless communication systems is well established. Numerous tests conducted in the past several years have proven the potential for high bandwidth, low bit error rate optical communication in the underwater domain. It has been shown that laser communication using 400-600 nm wavelength beams minimizes absorption by water [2-6]. Using beams in the specified range, it is possible to establish a communication link underwater with extremely high fidelity and data rate (rates into the Gbps range have been realized) [3-4].

However, laser propagation through water is generally not well understood. The underwater channel is inherently more unpredictable than the free space channel, in that factors such as turbidity, salinity and temperature can significantly affect the propagation of the beam [7-8]. Additionally, while beam propagation has been successfully modeled using Monte Carlo simulation (see [9] for example), it has not, as to our knowledge yet been fit to a probabilistic model.

Knowledge of the probability density function that describes beam propagation in water would provide valuable insight into the performance of an underwater wireless optical link. For a free space link, the probability density function is an important tool for estimating bit error rate and performance. In this project, we attempt to fit various probability density function models to an underwater link, and investigate the effect that water temperature have on these models.

We constructed a lab emulator for a short-range underwater optical wireless link, simply consisting of a laser transmitter, CMOS camera, and 1 meter long plexiglass water tank. Specific knowledge of the volume of water present allows for precise control of factors such as salinity and turbidity, and a several submerged electric heaters allow precise control on the water temperature. Using the emulator, we can with reasonable accuracy model the water in a desired location around the world given characteristics such as salinity, turbidity and average temperature, and gain valuable insight into some of the scaled effects of the actual environment while in a controlled laboratory setting.

Ocean Sensing and Monitoring VIII, edited by Weilin W. Hou, Robert A. Arnone, Proc. of SPIE Vol. 9827, 98270D · © 2016 SPIE · CCC code: 0277-786X/16/$18 · doi: 10.1117/12.2229814

We discuss the accuracy of several probability density functions in the underwater environment; namely, the Weibull and exponentiated Weibull distributions. To prove the utility of these measurements in the real world, we conducted a series of tests on brackish water gathered at the United States Naval Academy, from the Severn River. We first took measurements of the various characteristics of the collected Severn water, specifically salinity and temperature gradient. Using the emulator, we then recreated these conditions in the laboratory, and compared them to the PDF models.

In our analysis, we made use of several parameters normally applied to probabilistic analysis of optical beam propagation in the free space domain. The main goal of this project was to determine if it was possible to alter these parameters to more closely fit the underwater channel. In order to accomplish this, we used a nonlinear optimization technique reliant on MATLAB to generate best fit parameters for our experimental data.

Knowledge of the probabilistic model for underwater optical beam propagation would provide significant insight into the performance of an underwater optical link. The ability to adjust the model to fit various salinities and turbidities from water around the globe may facilitate the efficient use of underwater optical communications more universally.

Additionally, the United States Navy is interested in this area of research. It envisions underwater optical communications systems being used by combat and salvage divers to communicate wirelessly and covertly while conducting operations underneath the surface. The currently feasible range for such devices is likely <100m, and this range is perfectly applicable to diver-to-diver communication. In the future, the Navy also hopes to develop optical communications for Unmanned Underwater Vehicle (UUV) control and wireless communication between submarines and surface ships. Knowledge of the probability density function associated with the underwater channel would greatly aid in the development of these technologies.

In a previous paper, we explored the applicability of various Free Space optical turbulence models developed by Barrios et al. [12] to the underwater channel. In this paper, we aim to extend that research by optimizing the models to specifically fit the underwater channel, given experimental data.

2. Theoretical Background

The main goal of this paper is relate measurable characteristics of the underwater channel to the shape parameters used to generate the PDFs. One of the most important of these is scintillation index. In Free Space Optics, scintillation index is a mathematical description of the normalized atmospheric variations that affect the propagation of the beam, resulting from optical turbulence [10]. Such variations in free space are primarily the result of temperature variations in the atmosphere which cause small fluctuations in the index of refraction and subsequently cause optical turbulence. Mathematically, scintillation index is defined by

$$\sigma_I^2 = \frac{< I^2 > - < I >^2}{< I >^2} = \frac{< I^2 >}{< I >^2} - 1 \qquad (1)$$

Where *I* denotes irradiance of the beam and < > denotes an ensemble average. How we measure irradiance over time will be discussed in depth in section 3.

Research has proven that certain wavelengths of light propagate much more effectively through water than others. Specifically, lasers in the blue-green portion of the spectrum (470 – 570 nm) experience the smallest amount of scattering and absorption in water. For this paper, a 632 nm beam generated by a 1.5 mW HeNe laser was used.

2.1 Probability Density Functions

Until fairly recently, the Weibull probability distribution was not widely used in description of the FSO channel. A 2013 paper first introduced it as a viable alternative to the generally accepted Lognormal and gamma gamma distribution [12]. It showed that the Weibull and exponentiated Weibull distributions offer an excellent fit for data under all turbulence conditions in the free space domain. The exponentiated Weibull was proven to show a near-perfect fit in the tail for irradiance data [12]. This paper aims to investigate whether these distributions and the shape parameters associated with them can be refined using a non-linear optimization method to more closely fit the underwater channel. The Weibull probability density function is defined by

$$p_I(I) = \frac{\beta}{\eta}\left(\frac{I}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{I}{\eta}\right)\right]^{\beta} \quad (3)$$

where β and η are constants related to scintillation index, defined by

$$\sigma_I^2 \approx \beta^{-6/11} \quad (4)$$

$$\eta = \frac{1}{\Gamma(1+\frac{1}{\beta})} \quad (5)$$

where Γ represents the gamma function. β and η are both shape parameters used to describe beam propagation in the free space domain [12].

The exponentiated Weibull distribution, also introduced by Barrios and Dios, may prove to be a unique and interesting model for the underwater channel. While complex, it may be useful in the inherently unpredictable underwater environment. The probability density function is defined by

$$p_I(I) = \frac{\beta}{\eta}\left(\frac{I}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{I}{\eta}\right)\right]^{\beta} \left\{1-\exp\left[-\left(\frac{I}{\eta}\right)\right]^{\beta}\right\}^{\alpha-1} \quad (6)$$

α, β and *η* are shape parameters [12] calculated using a fitting procedure to simulate data and relate *α, β* and *η* to the scintillation index. These parameters are defined by

$$\alpha \cong \frac{7.220\sigma_I^{2/3}}{\Gamma(2.487\sigma_I^{2/6}-0.104)} \quad (7)$$

$$\beta = 1.012(\alpha\sigma_I^2)^{-13/25} + 0.142 \quad (8)$$

$$\eta = \frac{1}{\alpha\Gamma(1+\frac{1}{\beta})g_1(\alpha,\beta)} \quad (9)$$

In the calculation of η, a separate function, *g*, is used, which is defined by

$$g_n(\alpha,\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k\Gamma(\alpha)}{k!\,(k+1)^{1+\frac{n}{\beta}}\Gamma(\alpha-1)} \quad (10)$$

which, converges very quickly. For data processing purposes, we use 10 terms to approximate g. The added shape parameter α controls lower tail steepness [12]. As α approaches 1, the exponentiated Weibull distribution becomes the standard Weibull distribution.

2.2 Least Squared Error for Assessing Fit

In assessing the various probabilistic models introduced above, we use least squared error (LSE) analysis to determine deviation of the data from the expected curve. In this report, we use both overall LSE of the curve and LSE in the tail. For communication systems, LSE in the tail of the curve is the determining factor in the accuracy of a probability model in predicting bit-error rate.

LSE for this project was calculated by using the value at the histogram bin center to calculate the probability of the PDF at the specified intensity value, and calculating the difference between the data value and the value of the PDF model at that point. This difference was then squared and summed across the number of bins to give the total LSE. For the tail LSE calculation, the LSE for the first 10 bins on the left side of the PDF function was calculated to show the quality of fit in the tail as compared to the quality of overall fit.

In all figures shown in Section 4, red dots indicate the values for the data histogram bin centers, and the curves indicates the calculated Probability Density Functions.

3. Experimental Methodology

In order to gather irradiance data in a coherent and reproducible way, a simple lab bench testbed was constructed wherein various characteristics of water could be measured and controlled. A 1 meter long plexiglass tank was used, so that the volume of water used could be precisely measured. The laser, a 1.5 mW HeNe laser was placed at one end of the tank, and a CMOS camera (1280x1024 pixels) was placed at the other. Additionally, a red bandpass filter was affixed to the camera in order to minimize the effects of ambient light on the measurements, and a neutral density filter was also used to protect the camera from pixel saturation.

Another of the major goals of this project was to investigate the effect of temperature on beam propagation using probabilistic models. To achieve such gradients, several electric heaters normally used in kitchen applications were placed in the tank at 25 cm intervals. They were powered using variable DC sources, so that the voltage supplied, and therefore the temperature of the water, could be precisely controlled. This ensured even distribution of warm water along the beam, to simulate minute temperature changes in water. Thermocouples were affixed near each heating element in order to measure the temperature along the beam. Figures 1 and 2 show the testbed setup.

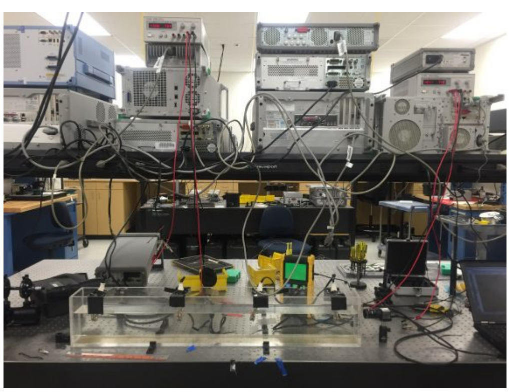


Figure 2: Testbed Setup

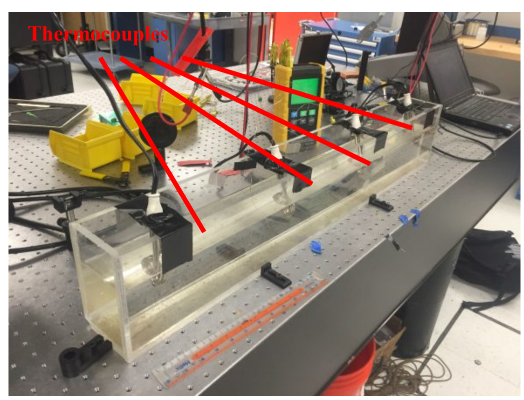


Figure 1: Testbed Setup

3.1 Shape Parameter Optimization Methodology

In order to experimentally determine shape parameters for the previously discussed probability density functions, multiple experimental data trials of approximately 20 seconds at approximately 200 frames per second, using brackish water from the Severn River gathered at the US Naval Academy, were performed in the underwater channel emulator described above. From these experimental trials, the scintillation index was determined and a non-linear curve fitting routine was run to find the Weibull and exponentiated Weibull parameters that best fit these probability density functions to the experimentally collected data.

From here, each parameter (α, β and η) was plotted as a function of scintillation index, and a best fit curve was generated for these plots. We refer to this method as the "1st order fit." This fitting method, which generates a general curve that gives each shape parameter as a function of scintillation index, is not related to the shape parameters generated by Barrios et al.

Next, in order to gather insight over a larger range of scintillation index, more consistent with the findings of Barrios et al, a non-linear fit routine was again used to modify the shape parameters alpha, beta and eta. Specifically, the numerical coefficients used in the equations heuristically developed by Barrios et al. were optimized and fit to the experimental data. We refer to this fitting method as our "2nd order fit."

Finally, we assessed how the shape parameters generated by our 1st and 2nd order fitting methods affected the accuracy of the probability density functions, using Least Squared Error analysis as discussed above. The results generated for the shape parameters, and the Least Squared Error analysis are presented and discussed further in the next section.

4. Results and Discussion

To establish a baseline for standard water, we take a series of samples with simple distilled water. We also subtract ambient light from each trial, obtained by taking the average of the 28x28 pixel box with no laser illumination. The same 28x28 pixel box is used in each of the subsequent data collection trials. The following figure is a time series of irradiance over 20 seconds for distilled water. Scintillation index here is almost negligible, on the order of 1e-5. The time series for this trial is shown in figure 3.

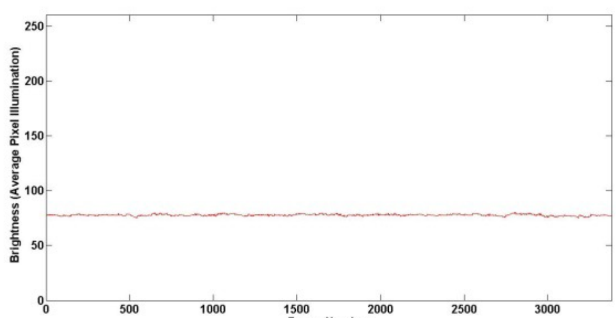


Figure 3: Optical Power Time Series

The first iteration of this project was to assess the performance of the Free Space models discussed above in the underwater channel. The average irradiance values over time were plotted in a histogram, and goodness of fit based on least square error analysis was assessed [13]. Figure 4 shows the result of this process using the Free Space Optics equations described in Section 2.

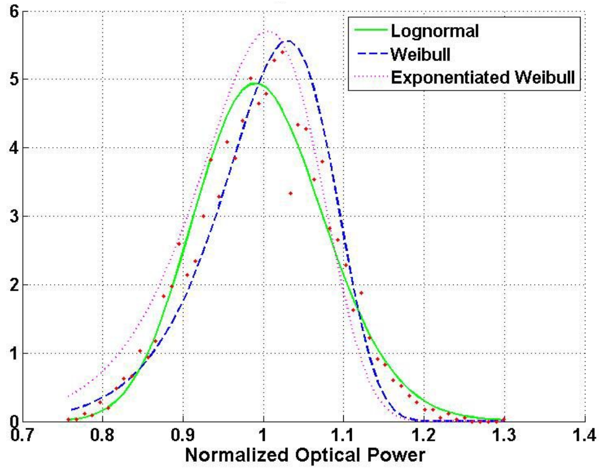


Figure 4: Histogram PDFs for FSO models Underwater

In order to optimize these parameters, a large quantity of samples were taken over a range of scintillation indices. Then a non-linear optimization method was used to identify the parameter for the best fit PDF for each value of scintillation index. These values were fit to a curve, to generate an equation for each parameter specific to the underwater channel. The results of this 1st order analysis for the Weibull distribution are shown below in Figures 1.

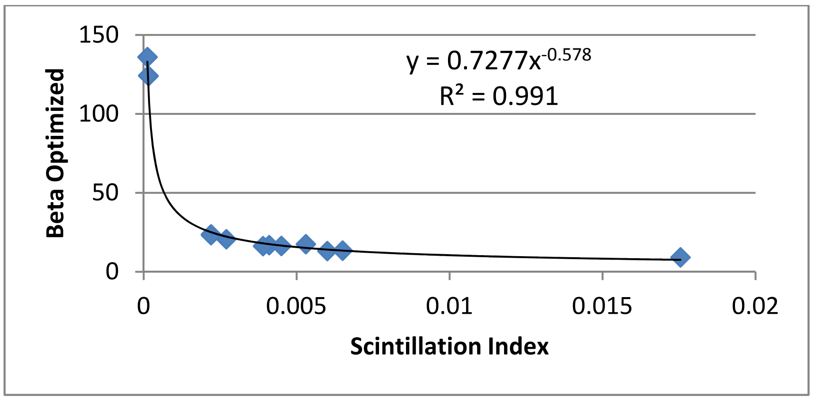


Figure 5: Weibull Beta Optimized - 1st Order

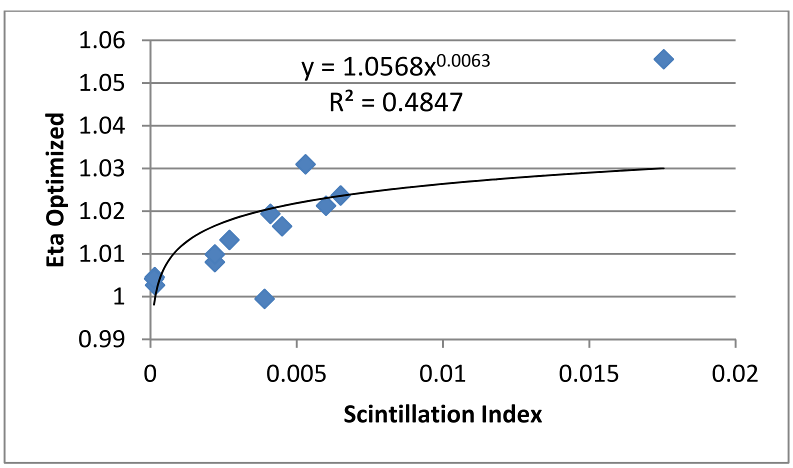


Figure 6: Weibull Eta Optimized - 2nd Order

The same analysis for the shape parameters of the Exponentiated Weibull Distribution are shown in figures 7, 8 and 9.

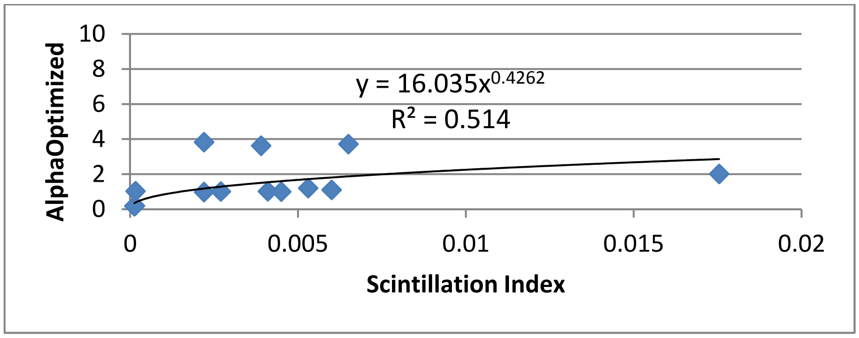


Figure 7: Exponeniated Weibull Alpha Optimized - 2nd Order

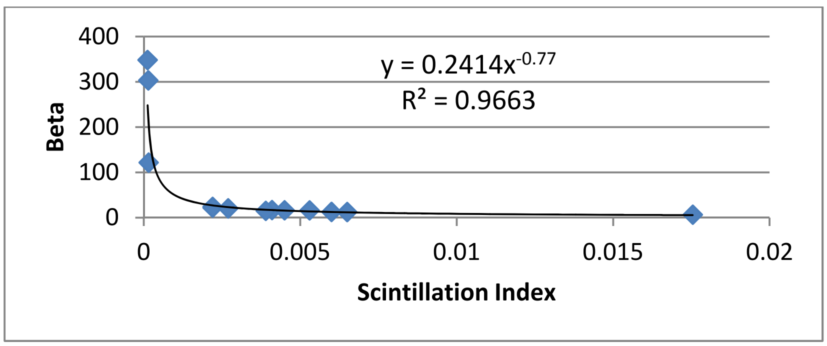


Figure 8: Exponentiated Weibull Beta Optimized - 2nd Order

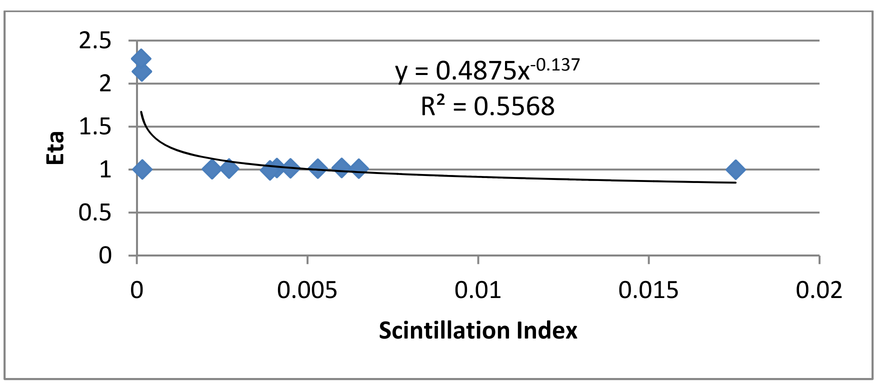


Figure 9: Exponentiated Weibull Eta Optimized - 2nd Order

In further attempting to refine the fit of the shape parameter curves, another iteration of non-linear curve fitting was performed on the refined shape parameters. In this iteration, the coefficients used by Barrios et al. in calculating the shape paramters α and β were optimized using the previously optimized shape parameters and associated scintillation index. Specifically, the coefficients in equations 7 and 8 were optimized; for example, the first coefficient in Eq. 7 was optimized from a value of 7.220 to 4.153 The new coefficients are shown in the equations

11 and 12. Note that, the exponents used by Barrios et al. were not considered in our analysis. Due to the iterative nature of our optimization method, only one aspect of the equations was changed.

$$\alpha \cong \frac{4.1543\sigma^{2/3}}{\Gamma(12.1717\sigma^{\frac{2}{6}} - 8.1447)} \qquad (11)$$

$$\beta = 2.9972(\alpha\sigma^2)^{-13/25} - 30.8311 \qquad (12)$$

The final step of our investigation was to compare the accuracy of our non-linearly optimized shape parameters specific to the underwater channel to the accuracy of the shape parameters used by Barrios et al. for free space. Figure 10 shows the previously analyzed Exponentiated Weibull distribution for the original Barrios et al. equations, with Least Squared Error shown. The data used is the average of several 20 second data trials using brackish water from the Severn River. The temperature gradient along the beam was roughly 0.025°C per centimeter.

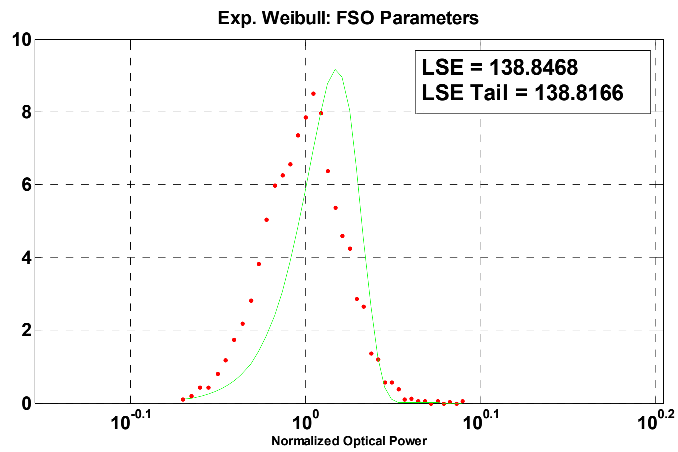


Figure 10: FSO Fit for Severn Water w/ Gradient

Figure 11 shows the same histogram analysis for the 1st order fit.

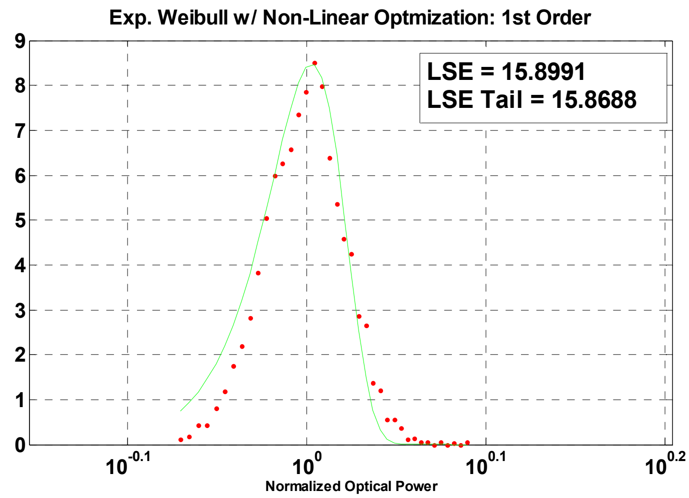
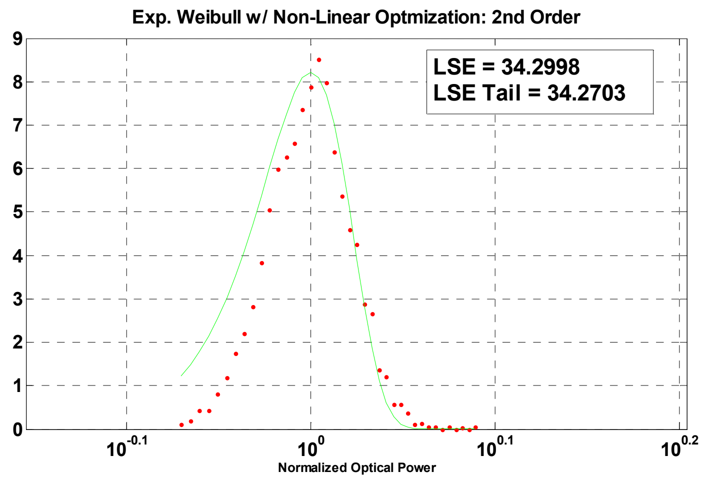


Figure 11: 1st Order Fit for Severn Water w/ Gradient

Figure 12 shows the analysis for the 2nd order fit.



5. Conclusions

On examination of the probability density functions using the 1st and 2nd order non-linearly optimized shape parameters, it is clear that the accuracy of the PDF is significantly improved. Specifically, the 1st order shape parameters showed a LSE of 15.8991 and a LSE of 15.8688 in the tail. The 2nd order shape parameters showed a LSE of 34.2998 and a LSE of 34.2703 in the tail. Both were several times lower as compared to the LSE shown in the original PDF used for Free Space, roughly 138 in both the body and the tail. It is also clear that in this case, the 1st order fit is more effective than the second order fit. This will be investigated further in future work.

Based on the results of this project, it is reasonable to conclude that non-linear curve fitting techniques to optimize probability density function shape parameters are useful for conducting PDF analysis in the underwater domain. Further work promises to yield equations that provide a more complete picture of the underwater channel, and additional applicability to underwater channels in different parts of the world.

6. References

[1] A. K. Vikrant and R. Jha, "Comparison of Underwater Laser Communication System with Underwater Acoustic Sensor Network," *International Journal of Scientific and Engineering Research,* 2012.

[2] W. T. F. and K. Sherman, "Role of Blue/Green Laser Systems in Strategic Submarine Communications," *IEEE Transactions on Communications,* vol. 28, no. 9, 1980.

[3] F. Hanson and S. Radic, "High Bandwidth Underwater Optical Communication," *Applied Optics,* vol. 47, no. 2, pp. 277-283, 2008.

[4] T. Scholz, "Using Laser Communication Above Water and Underwater," *Sea Technology,* May 2011.

[5] J. Giles and I. Bankman, "Underwater Optical Communications: Basic Design Considerations," in *Military Communications Conference 2005*, 2005.

[6] B. Cochenour, L. Mullen and A. Laux, "Spatial and Temporal Dispersion in High Bandwidth Underwater Laser Communication Links," *Military Communications Conference*, *2008*, 2008.

[7] F. Hanson and M. Lasher, "Effects of Underwater Turbulence on Laser Beam Propagation and Coupling into Single-mide Optical Fiber," *Applied Optics,* vol. 49, no. 16, 2010.

[8] W. Lu, L. Liu and J. Sun, "Influence of Temperature and Salinity Fluctuations on Propagation Behavior of Partially Coherent Beams in Oceanic Turbulence," *Journal of Optics: Pure and Applied Optics,* vol. 8, pp. 1052-1058, 2006.

[9] C. Gabriel, M.-A. Khalighi, S. Bourennane, P. Leon and V. Riguad, "Monte Carlo Based Channel Characterization for Underwater Optical Communication Systems," *Optical Communications Review,* vol. 5, no. 1, 2013.

[10] L. Andrews and R. Phillips, Laser Beam Propagation through Random Media, Bellingham: SPIE, 2005.

[11] J. Aitchison and J. Brown, The Lognormal Distribution. Cambridge: At the Univ. Press, 1963.

[12] R. Barrios and F. Dios, "Exponentiated Weibull Fading Model for Free Space Optical Links with Partially Coherent Beams under Aperture Averaging," *Optical Engineering,* vol. 52, no. 4, 2013.

[13] M. Bernotas and C. Nelson, "Probability density function analysis for optimization of underwater optical communications systems," *OCEANS 2015 - MTS/IEEE Washington*, Washington, DC, 2015, pp. 1-8.