# Practical Exam: Airy Disks

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## Introduction

For this practical exam, we were required to research Airy Disks, predict the locations of the disks based on three different aperture diameters, and photograph the disks to compare predicted locations with experimental locations. Airy Disks are the formation of light patterns that occur when light passes through a small aperture. Study of these patterns has led to the formation of mathematical models that predict the distance of maxima and minima of the disks from the center of light; in our case, light from a red laser beam. A quick search of the Internet revealed the relatively simple formula for this prediction. Perhaps more interesting are the patterns developed from two side-by-side apertures or three apertures in a triangle formation, which we were able to observe and photograph; examples of which are included in this report. Ultimately, the mathematical model did not align with the experimental results, and further experimentation would be required to discover the reason.

## Mathematical Prediction

The mathematical model for the locations of the Airy Disks is relatively simple. Using the small angle approximation for sine, the wavelength of the laser light $\lambda$, the distance of the aperture from the target where the beam was photographed $D$, the diameter of the apertures $d$, and experimental constants for each minima and maxima $m$; we can predict the distance of the minima or maxima $y$ from the center of the beam:

$$y = \frac{Dm\lambda}{d}\,. \qquad (1)$$

For each of the three apertures, three minima and maxima were predicted. The source used was a red laser light with wavelength 632.8 nm, whose beam was immediately expanded and passed through the aperture. A camera was set up 4.5 m downrange, with 0.9 power intensity filter and a red light filter. The results of these predictions are in the following tables.

| Aperture d = 0.7366 mm | | |
|---|---|---|
| Number of Minima or Maxima | Minima Distance (mm) | Maxima Distance (mm) |
| 1 | 4.7164 | 6.3207 |
| 2 | 8.6325 | 10.3567 |
| 3 | 12.5177 | 14.2651 |

***Table 1 Predictions for distance of minima and maxima from beam center for the small aperture***

| Aperture d = 1.0414 mm | | |
|---|---|---|
| Number of Minima or Maxima | Minima Distance (mm) | Maxima Distance (mm) |
| 1 | 3.3360 | 4.4707 |
| 2 | 6.1059 | 7.3254 |
| 3 | 8.8540 | 10.0899 |

***Table 2 Predictions for distances of minima and maxima from the beam center for the medium aperture***

| Aperture d = 1.9812 mm | | |
|---|---|---|
| Number of Minima or Maxima | Minima Distance (mm) | Maxima Distance (mm) |
| 1 | 1.7535 | 2.3500 |
| 2 | 3.2095 | 3.8506 |
| 3 | 4.6540 | 5.3037 |

***Table 3 Predictions for the distances of minima and maxima from the beam center for the large aperture***

A recognizable trend occurs with an increase in aperture size. As the aperture diameter increases, the distances of the disks from the center of the beam are reduced. Further research unveiled that this trend is due to the wave properties of light; that a smaller aperture closer in size to the wavelength of the light causes more spreading than an aperture that relatively larger than the wavelength of the light.

## Setup

A portion of this practical exam was setting up the experiment. To obtain the most accurate results, the laser was directed onto the camera sensor. Knowing the size of sensor and each pixel, an accurate measure could be taken of the minima and maxima. The images were captured by moving the camera perpendicular to the beam propagation path for each maxima of the Airy Disk. For the imaging software, the same settings could not be used for each image of an aperture because the second and third disks were not visible. This did not affect the intensity measurement on the sensor, however. In addition to directing the beam onto the sensor, we photographed the beam and subsequent Airy Disks on a white target with a 17 mm lens, and again with an iPhone camera to best represent what the eye sees.

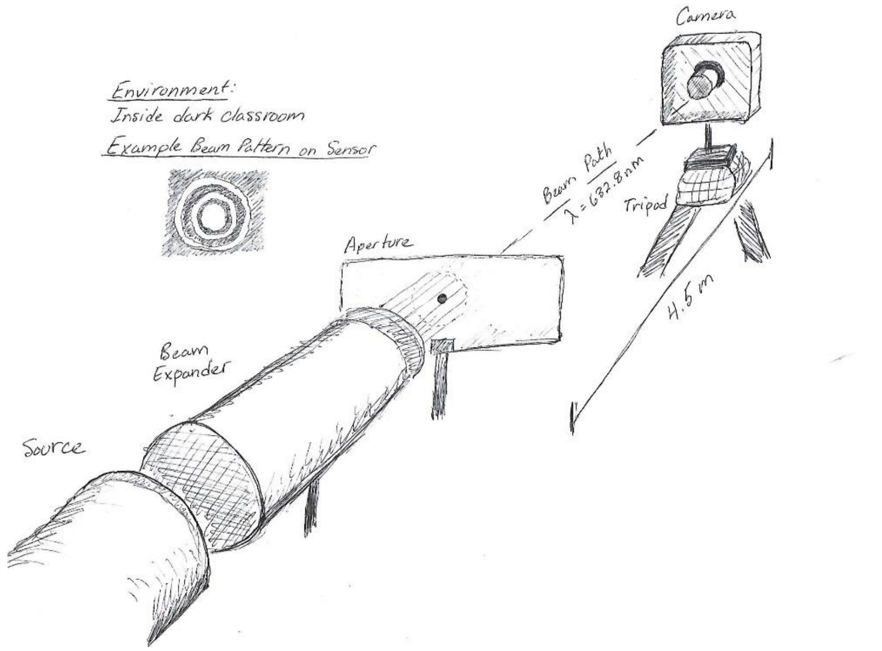


***Figure 1 Sketch of the experiment setup***

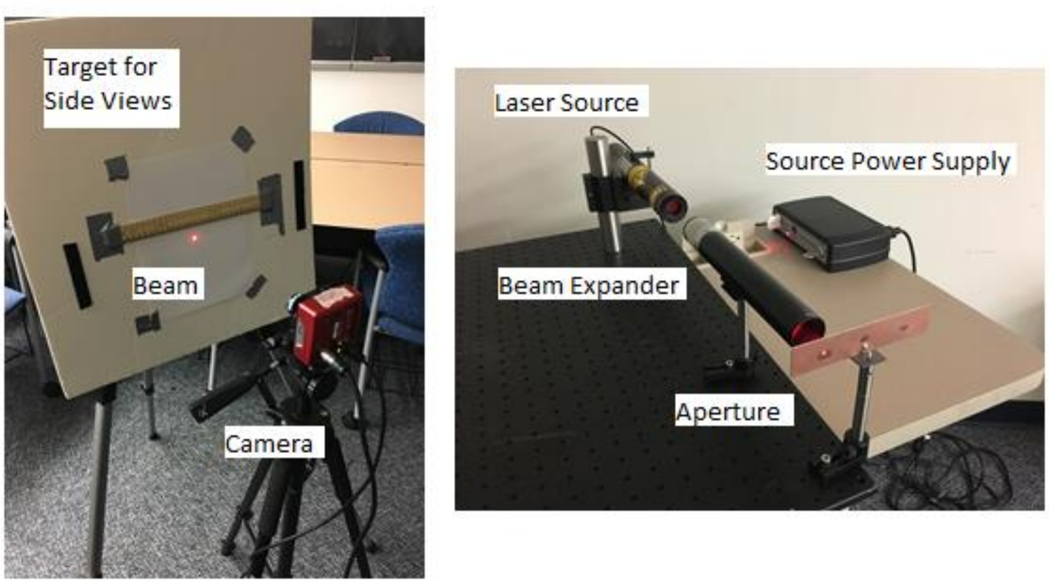


***Figure 2 Setup of camera for side images, laser source and expander with aperture, and image of apertures used in the experiment***

## Experimental Results and Analysis

Once the pictures were taken, the .tif files from the camera software were processed in MATLAB. For each aperture, a picture of the center beam, the first maxima, second maxima, and third maxima were taken. Each picture of a maxima contained the surrounding minima, which was how the measurements were standardized. To calculate the minima and maxima, the 'findpeaks' function was used. The results are shown in Tables 4-6.

| Aperture d = 0.7366 mm | | | | |
|---|---|---|---|---|
| Number of Minima or Maxima | Predicted Minima Distance (mm) | Experimental Minima Distance (mm) | Predicted Maxima Distance (mm) | Experimental Maxima Distance (mm) |
| 1 | 4.7164 | 3.5960 | 6.3207 | 4.6838 |
| 2 | 8.6325 | 6.4524 | 10.3567 | 7.8214 |
| 3 | 12.5177 | 9.3532 | 14.2651 | 10.6778 |

***Table 4 Results of experiment with mathematical predictions for small aperture***

| Aperture d = 1.0414 mm | | | | |
|---|---|---|---|---|
| Number of Minima or Maxima | Predicted Minima Distance (mm) | Experimental Minima Distance (mm) | Predicted Maxima Distance (mm) | Experimental Maxima Distance (mm) |
| 1 | 3.3360 | 2.7338 | 4.4707 | 4.0140 |
| 2 | 6.1059 | 5.4940 | 7.3254 | 6.5596 |
| 3 | 8.8540 | 8.1580 | 10.0899 | 9.3124 |

***Table 5 Results of experiment with mathematical predictions for medium aperture***

| Aperture d = 1.9812 mm | | | | |
|---|---|---|---|---|
| Number of Minima or Maxima | Predicted Minima Distance (mm) | Experimental Minima Distance (mm) | Predicted Maxima Distance (mm) | Experimental Maxima Distance (mm) |
| 1 | 1.7535 | 2.9230 | 2.3500 | 3.8998 |
| 2 | 3.2095 | 5.8830 | 3.8506 | 6.9782 |
| 3 | 4.6540 | 7.8736 | 5.3037 | 10.1824 |

***Table 6 Results of experiment with mathematical predictions for large aperture***

Overall, an interesting trend is observed from the results. For the small aperture, the prediction overshoots the values for the experimental results. The medium aperture has a smaller deviation, though the prediction again overshoots the experimental results. For the large aperture, the prediction undershoots the experimental data by nearly half for each value. It would be interesting to see these results replicated at a farther and shorter distance between the aperture and the camera sensor. Figure 2 shows an example of how the locations of the distances of the rings from the center were calculated.

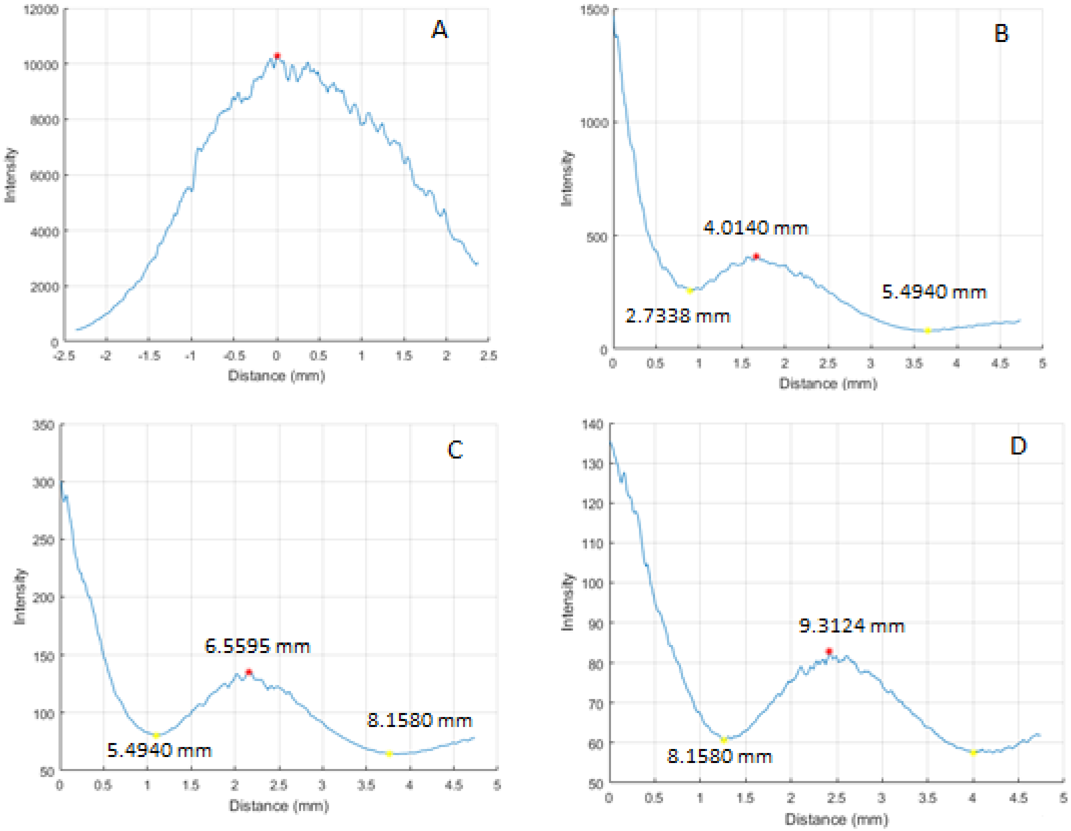


***Figure 3 Using 'findpeaks' to measure distances of maxima and minima from the center of the beam for the medium aperture, d = 1.0414 mm. The locations of each point were recorded, and differences between the minima and maxima were taken to find total distance from the center. Figure 2.A is the center maxima; 2.B is m=1 minima, m=1 maxima, and m=2 minima; 2.C is m=2 minima, m=2 maxima, and m=3 minima; 2.D is m=3 minima, m=3 maxima, and m=4 minima. Note that the distances on the axis do not correlate with the actual distance from the center maxima, though the labeled distances on the lines do correlate. Also, the y-axis range changes for each portion to best show the quality of each maxima.***

Visually, the Airy Disk pattern are pleasing to the eye. For a larger aperture, the disks are noticeably finer and closer together; and for the smaller aperture, the rings are wider and the edges are less sharp and almost appear out of focus. Pictures of the rings as the eye would see them are in Figure 3.

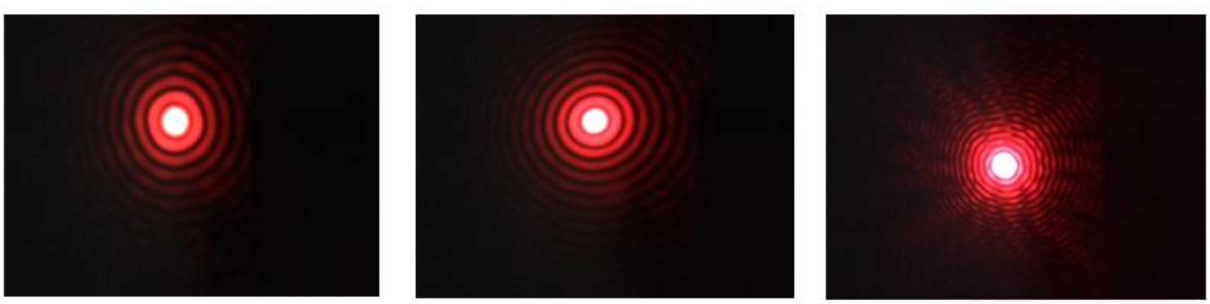

***Figure 4 Pictures taken of Airy Disks, left to right is 0.7366 mm, 1.0414 mm, and 1.9812 mm in aperture diameter.***

As the camera views the images from the side with a 17 mm lens, the images are less exciting though the intensity gradient is more clearly defined. The intensity of the outer rings is much less than that of the center beam or m = 1 and m = 2 ring.

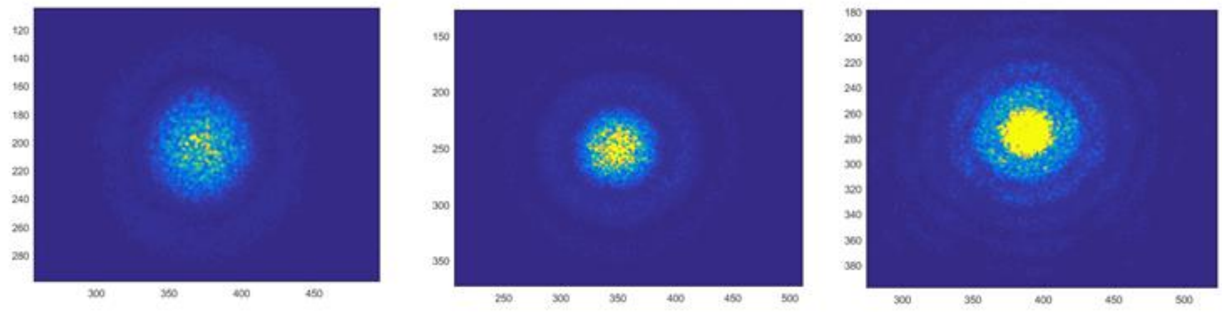

***Figure 5 Camera views highlighting intensity, from 0.7366 mm, 1.0414 mm, and 1.9812 mm left to right. The values of the axes are in pixels, which serve only to document the relative size of each image.***

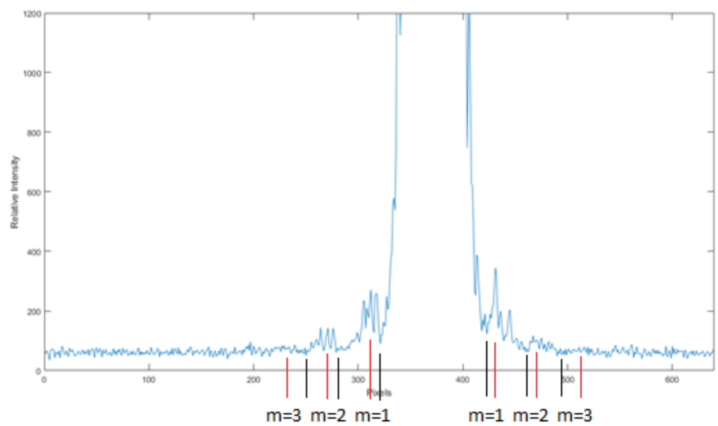


***Figure 6 Cross section of relative intensity of d = 0.7366 mm aperture. Red lines indicate a maxima and black lines indicate a minima. The center lobe of the beam was cut off to better view the maxima and minima.***

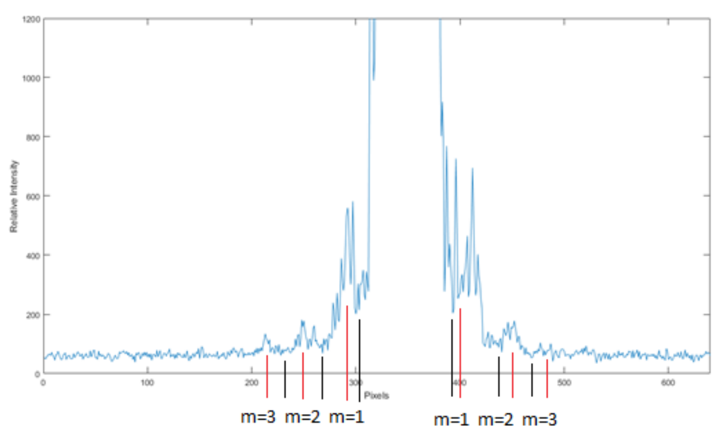


***Figure 7 Cross section of relative intensity of d = 1.0414 mm aperture. Red lines indicate a maxima and black lines indicate a minima. The center lobe of the beam was cut off to better view the maxima and minima.***

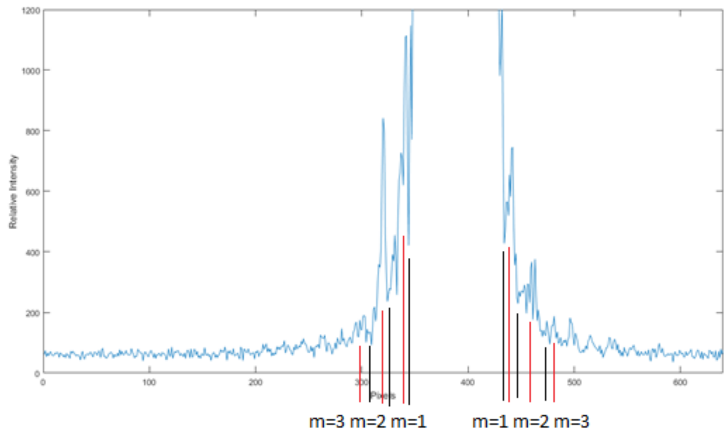


***Figure 8 Cross section of the relative intensity of the d = 1.9812 mm aperture. Red lines indicate a maxima and black lines indicate a minima. The center lobe of the beam was cut off to better view the maxima and minima.***

With the camera set up such that the beam was directly impacting the sensor, the relative intensity of the light was measured. The camera used has $2^{14}$ values to measure light, allowing precise measurement of light intensity from the beam. Using experimental coefficients for each relative intensity of a given m-value, the intensity of the disk can be compared to the intensity of the central beam. The comparison between the predicted relative intensities and measured relative intensities are shown in Figure 5.

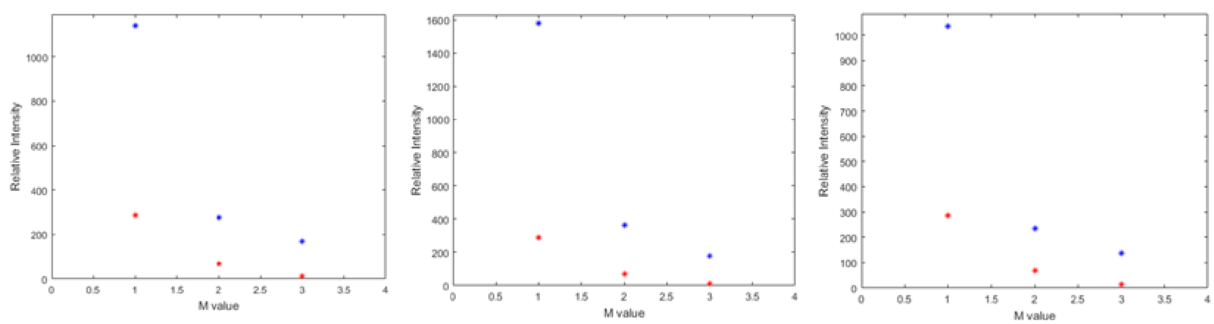


***Figure 9 Comparison of relative intensities, predicted and measured. From left to right, the small to large aperture. Predicted relative intensities are in red and actual relative intensities are in blue.***

To note from the graphs of the relative intensities, the measured intensities are larger than the predicted intensities. However, all follow an exponential decrease pattern as the m-value increases. Each measured relative intensity was found by dividing the intensities of the maxima by the maximum intensity of the central portion of the beam.

For further experimentation, we viewed the patterns of the beam passed through two apertures side-by-side and a triangular formation of apertures. No mathematical model exists for these patterns due to their complication. However, they are interesting to view, shown in Figures 6 and 7.

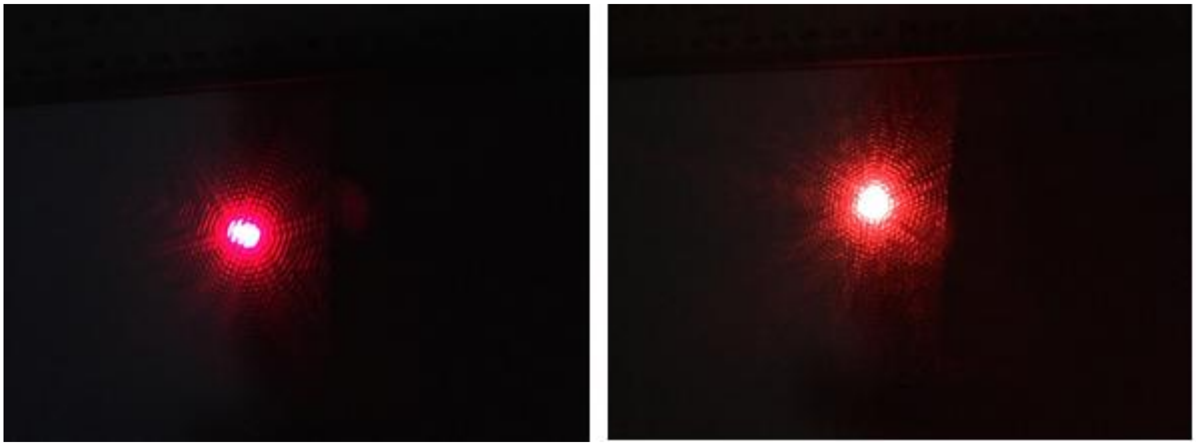

***Figure 10 Images of the pattern for two apertures side-by-side and three apertures in triangular formation as the eye would see them, left to right.***

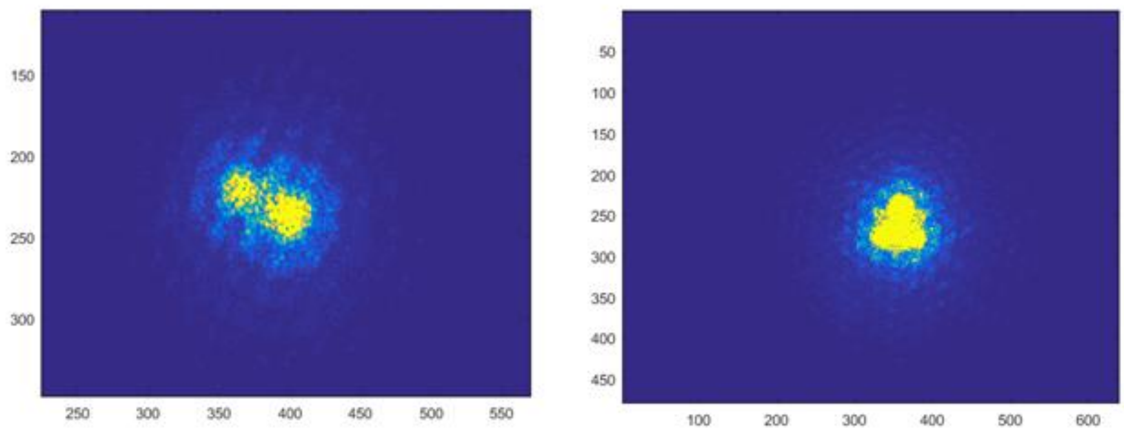


***Figure 11 Images of the two and three aperture patterns enhanced to view intensity. The axes are in units of pixels and serve simply to compare relative sizes of the patterns.***

## Conclusion

Our experiment with Airy Disks highlighted a few properties that are interesting to note. One is how the size of the aperture affects the disks and their intensity. As the aperture widens, the disks become more fine and packed together. This is most likely due to the wave properties of light as it passes through an aperture and refracts away. Compared to other images of Airy Disks on the Internet, we succeeded in replicating the patterns that many others acquired. Further investigation might reveal why the mathematical model and experimental results do not align, but a hypothesis might involve the use of a small angle approximation in the mathematical model. There could also be some error occurring when the values for the cross section of each maxima were created, leading to error in actual location of the areas of interest. In comparison, the measured relative intensity of the first three disks follows the exponential decrease pattern that the predicted intensity coefficients hypothesize, though the measurements are not of the same magnitude even after normalization.

# Appendix

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```
% This code predicts minima and maxima of airy disks from center of beam,
% imports .tif images of beams, analyzes those images by measurement and
% beam intensity, and graphically displays results.
```

## Airy Disk maxima and minima prediction

```
lambda = 632.8e-6; % wavelength of laser light, nm
d1 = 0.7366e-3; % single-hole aperture sizes, mm
d2 = 1.0414e-3;
d3 = 1.9812e-3;
D = 4.5; % distance from aperture to target, where Airy disks measured
minim = [1.220 2.233 3.238]; % vector of minima m values
maxim = [1.635 2.679 3.69]; % vactor of maxima m values
relint = [0.0175 0.0042 0.00078]; % vector of predicted maxima relative intensities to beam

% rows are predicted maxima/minima values for an aperture, apertures
% increase down columns
ymax = [maxim.*lambda*D/d1; maxim.*lambda*D/d2; maxim.*lambda*D/d3];
ymin = [minim.*lambda*D/d1; minim.*lambda*D/d2; minim.*lambda*D/d3];
```

## Reading in images for analysis

```
InfoImage = imfinfo('1holeLarge0.tif'); % getting image info
Image = InfoImage(1).Width; % variable for width of image
nImage = InfoImage(1).Height; % variable for height of image

im0 = imread('1holeLarge0.tif');
im1 = imread('1holeLarge1.tif');
im2 = imread('1holeLarge2.tif');
im3 = imread('1holeLarge3.tif');% reading individual pixel values and assigning variable for
array

% im0 = imread('1holeMedium0.tif');
% im1 = imread('1holeMedium1.tif');
% im2 = imread('1holeMedium2.tif');
% im3 = imread('1holeMedium3.tif');

% im0 = imread('1holeSmall0.tif');
% im1 = imread('1holeSmall1.tif');
% im2 = imread('1holeSmall2.tif');
% im3 = imread('1holeSmall3.tif');
```

## Pretty side view images

```
% im = imread('1holeLargeSideView.tif');
% im = imread('1holeMediumSideView.tif');
 im = imread('1holeSmallSideView.tif');
% im = imread('2holeLargeSideView.tif');
```

```
% im = imread('3holeLargeSideView.tif');
z = min(min(im)); % minimum pixel value
zz = max(max(im)); % maximum pixel value
aa = ceil(((im-z)./255));

figure,clf;
image(aa)
```

```
pixsz = 7.4e-6;
lngth = -319*pixsz:pixsz:320*pixsz;
avgcol = AvgCol(im0,Image);
avgcolsm = smooth(avgcol);
[pks,locs] = findpeaks(avgcolsm,lngth,'MinPeakDistance',4e-3);
centloc = locs - -319*pixsz; % -0.002227 for small % -0.001831 for medium % -319*pixsz for large
% beam center
halfbeam0 = zeros(2,320);
halfbeam0(1,:) = fliplr(avgcolsm(1:320));
halfbeam0(2,:) = pixsz:pixsz:320*pixsz;

figure(1),clf;
hold on
plot(lngth,avgcolsm)
plot(locs,pks,'r*')
xlabel('Distance (mm)')
ylabel('Intensity')
grid on

lngth = pixsz:pixsz:Image*pixsz; % converting pixels to length vector
avgcol = AvgCol(im1,Image); % average intensity vector for each column
avgcolsm = smooth(avgcol);
[pks,locs] = findpeaks(avgcolsm,lngth,'MinPeakDistance',1e-3); % finding peaks for maxima
[mins,locmin] = findpeaks(-1*avgcolsm,lngth,'MinPeakDistance',2e-3); % finding minima
dist12 = diff(locmin); % distance between minima
min1 = locmin(1); % isolating location of first minima in image
min21 = locmin(2); % isolating location of second minima in image
max1 = locs(2); % isolating maxima in image
minmax11 = max1 - min1; % difference between m = 1 minima and maxima
minmax12 = min21 - max1; % difference between m = 1 maxima and m = 2 minima
g1 = floor((locmin(2)-locmin(1))/pixsz); % size of needed array for continuous cross section
halfbeam1 = zeros(2,g1+1); % creating matrix for both lengths and intensities for continuous
cross section
halfbeam1(1,:) = avgcolsm(locmin(1)/pixsz:floor(locmin(2)/pixsz)); % populating continous cross
section matrix
halfbeam1(2,:) = halfbeam0(2,end) + lngth(locmin(1)/pixsz:floor(locmin(2)/pixsz));

% For m = 1,2 minima; m = 2 maxima
figure(2),clf;
hold on
plot(lngth,avgcolsm)
plot(locs(2),pks(2),'r*')
plot(locmin,-mins,'y*')
xlabel('Distance (mm)')
ylabel('Intensity')
grid on
```

```
avgcol = AvgCol(im2,Image);
avgcolsm = smooth(avgcol);
[pks,locs] = findpeaks(avgcolsm,lngth,'MinPeakDistance',2e-3);
[mins,locmin] = findpeaks(-1*avgcolsm,lngth,'MinPeakDistance',2.5e-3);
dist23 = diff(locmin);
max2 = locs(2);
min22 = locmin(1);
min31 = locmin(2);
minmax22 = min22 - max2;
minmax23 = min31-max2;
g2 = floor((locmin(2)-locmin(1))/pixsz);
halfbeam2 = zeros(2,g2+1);
halfbeam2(1,:) = avgcolsm(floor(locmin(1)/pixsz):floor(locmin(2)/pixsz));
halfbeam2(2,:) = halfbeam1(2,end) + lngth(floor(locmin(1)/pixsz):floor(locmin(2)/pixsz));

% For m = 2,3 minima, m = 2 maxima
figure(3),clf;
hold on
plot(lngth,avgcolsm)
plot(locs(2),pks(2),'r*')
plot(locmin,-mins,'y*')
xlabel('Distance (mm)')
ylabel('Intensity')
grid on

avgcol = AvgCol(im3,Image);
avgcolsm = smooth(avgcol);
[pks,locs] = findpeaks(avgcolsm,lngth,'MinPeakDistance',2e-3);
[mins,locmin] = findpeaks(-1*avgcolsm,lngth,'MinPeakDistance',2.5e-3);
dist34 = diff(locmin);
min32 = locmin(1);
min4 = locmin(2);
max3 = locs(2);
minmax3 = min32 - max3;
g3 = floor((locmin(2)-locmin(1))/pixsz);
halfbeam3 = zeros(2,g3+1);
halfbeam3(1,:) = avgcolsm(floor(locmin(1)/pixsz):floor(locmin(2)/pixsz));
halfbeam3(2,:) = halfbeam2(2,end) + lngth(floor(locmin(1)/pixsz):floor(locmin(2)/pixsz));

% For m = 3,4 minima; m = 3 maxima
figure(4),clf;
hold on
plot(lngth,avgcolsm)
plot(locs(2),pks(2),'r*')
plot(locmin,-mins,'y*')
xlabel('Distance (mm)')
ylabel('Intensity')
grid on

% This piece of code calculates the distances of the locations of the m = #
% maxima and minima, using the minima as reference points for each
% successive image.
minmaxT = [centloc+min1; centloc+max1; centloc+min21; centloc+min21-min22+max2; centloc+min21-
```

```
min22+min31; centloc+min21-min22+min31-min32+max3];

maxintens = max(max(im0));
predictintens = double(maxintens).*[0.0175 0.0042 0.00078]; % finding maximum intensity for the
beam, then multiplying by coefficients for m = 1,2,3
actintens = [max(max(im1(:,200:600))) max(max(im2(:,200:600))) max(max(im3(:,200:600)))];
compintens = cat(1,predictintens,actintens);
m = [1 2 3];

figure(5),clf;
grid MINOR
plot(m,predictintens,'r*',m,actintens,'b*')
ylabel('Relative Intensity')
xlabel('M value')
axis([0 3.5 0 max(actintens)+50])

% Making the concatenated vectors of beam center to third disk
halfbeam = zeros(2,321+g1+g2+g3+2);
halfbeam(1,:) = cat(2,halfbeam0(1,:),halfbeam1(1,:),halfbeam2(1,:),halfbeam3(1,:));
halfbeam(2,:) = cat(2,halfbeam0(2,:),halfbeam1(2,:),halfbeam2(2,:),halfbeam3(2,:));

figure(6),clf;
plot(halfbeam(2,:),halfbeam(1,:))
xlabel('Distance (mm)')
ylabel('Relative Intensity')
grid MINOR
```

---

```
imS = imread('1holeSmallSideView.tif');
imM = imread('1holeMediumSideView.tif');
imL = imread('1holeLargeSideView.tif');

figure(7),clf;
plot(imS(203,:))
ylabel('Relative Intensity')
xlabel('Pixels')
axis([0 640 0 1200])
figure(8),clf;
plot(imM(250,:))
ylabel('Relative Intensity')
xlabel('Pixels')
axis([0 640 0 1200])
figure(9),clf;
plot(imL(280,:))
ylabel('Relative Intensity')
xlabel('Pixels')
axis([0 640 0 1200])
```

## AvgCol Function

This function calculates the average value of a column of intensities from an image of a laser beam in order to plot the averages to find max intensities.

```
function [avgcol] = AvgCol(im,numpix)
    avgcol = zeros(numpix,1);
    for ii = 1:numpix
        avgcol(ii) = mean(im(:,ii));
    end
end
```

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