UNITED STATES NAVAL ACADEMY

Report

SM495

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Introduction

The purpose of this course is to provide background knowledge on laser beam propagation as well as conduct experiments oriented toward the ultimate goal of improving laser propagation through the maritime environment. Lasers can be a useful tool for remote sensing, communications, and as a weapon. However, because of the effects air turbulence has on a beam, using lasers in these areas has not yet been utilized to its full potential. Turbulence results in decreased intensity at the target in addition to increased scintillation (the amount a beam varies at a point). By possible manipulations to a beam at the source, we hope to achieve increased intensity at the target with decreased scintillation.

Motivation

As mentioned above, the purpose of this project is to produce a method that will allow laser beams to propagate through a maritime environment with minimal scintillation and little reduction in intensity. This new technique could then be applied as a way to communicate long distances or as a weapon. Recently, I discussed with a researcher how this type of system would be useful on a submarine. When a submarine surfaces, it is extremely vulnerable to attack by helicopters. Therefore, utilizing a laser targeting system would reduce its vulnerability so that the aircraft could be brought down more quickly. There are many more applications within the military, but this is just one example. Research in this area is being pursued fervently all around the country, and the world as well.

Personally, I was introduced to this research when my cousin, who attended the United States Naval Academy and graduated in 2010, encouraged me to take an Introduction to Laser Research course the second semester of my freshman year. I then became an assistant to a 1/C midshipman who was conducting his research in laser propagation and was introduced to Professor Avramov, Assistant Professor Nelson, and Professor Malek-Madani. Over the course of the semester, I became familiar with the equipment being used to conduct the research. Also, the classroom sessions introduced some of the mathematics involved with lasers and how it can be applied to real world scenarios. However, this research focuses on what happens when mathematics can no longer exactly predict what will happen because the environment through which the laser is propagating is not ideal. I am extremely interested in lasers because I can directly see the results of my research and how it will eventually be applied on a large scale.

Technically, this problem interests me because there are so many factors to consider. We have the tools to measure the atmospheric conditions and manipulate the beams. However, we only need to find the correct algorithm to allow the beam to propagate successfully. I hope that my time at the Academy will continue moving toward this goal and allow others to build off of my research to eventually reach a solution.

In my 1/C year, I would like to participate in the Trident Research Scholarship Program which would allow me to devote an entire year to an extensive research project in this field. In addition, I see the possibility of pursuing a master's degree after graduation from the Naval Academy and continuing my research.

Background Knowledge

In lectures with Professor Nelson, I am gaining an understanding of basic laser operations and the different theories which have led to the current understanding of the atomic orbitals and laser propagation.

History

1885

- Balmer discovers that the hydrogen atom has discrete energy levels

1900

- Planck develops his Black Body Radiation Distribution Law

$$p_{eq}(v) = \frac{8\pi h v^3}{c^3\left[exp\left(\frac{hv}{k_b T}\right)-1\right]} \quad \textbf{(1)}$$

  - p_{eq} is the equilibrium photon density
  - h is Planck's constant, 6.626E-34 kg m^2/s
  - c is the speed of light, 2.998E8 m/s
  - v is the frequency
  - k_B is the Boltzmann constant, 1.381E-23 kg^2 m^2/s^2 K
  - T is the temperature
- This equation predicts that at low frequencies, the radiated intensity from a blackbody will be relatively low. As frequency increases, so does radiation intensity until it reaches a peak and then begins to drop off, as shown in the graph below.

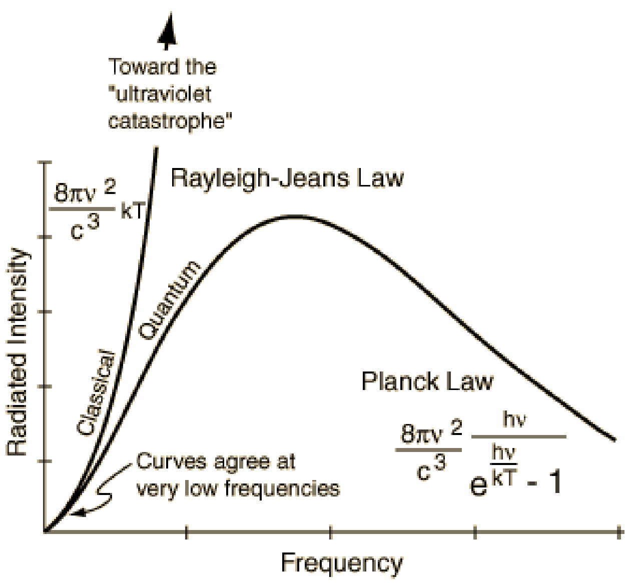


Figure 1: Planck's Black Body Radiation Distribution Law

http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html#c5

1905

- Einstein defines the photoelectric effect
  - $E = h\nu,\ E_1 - E_2 = h\nu$
    - "photons"

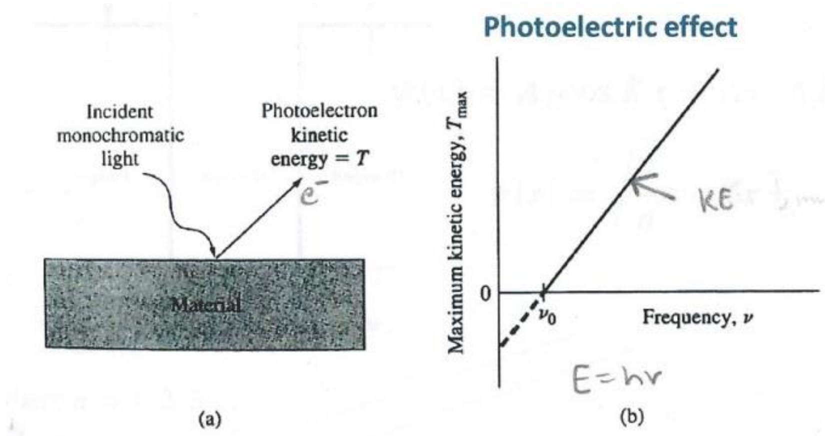


Figure 2: (**a**) The photoelectric effect (**b**) the maximum kinetic energy of the photoelectron as a function of incident frequency.

"Laser Fundamentals," 2nd edition, by William T. Silfvast, Cambridge, 2004.

<u>3 Key Ideas:</u>

1. The first key idea of quantum mechanics was given by Planck who described the atom as having discrete energy levels which produced photons of light, which he called quanta.

2. Next, Einstein identified the wave-particle duality of light, demonstrating how it possessed wave-like properties by diffraction but particle-like properties in delivery of discrete packets of energy, called photons.

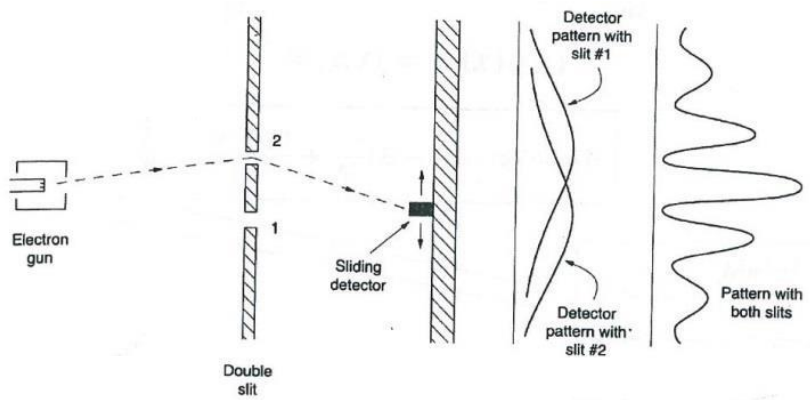


Figure 3: Double-slit diffraction pattern for electrons emitted from an electron gun, showing the pattern obtained when passing through each individual slit and then the pattern with both slits.

"Laser Fundamentals," 2nd edition, by William T. Silfvast, Cambridge, 2004.

3. Finally, Heisenberg outlined his Uncertainty Principle, stating that we could never know both how fast an electron is moving and where it is at the same time.

These 3 key ideas are the building blocks of quantum mechanics and are crucial to our understanding of light and lasers.

1913

- Bohr uses classical physics to define atomic orbitals
  - Assumes there is a mass at the center with electrons orbiting
  - $L = m_e vr = n\hbar = \frac{nh}{2\pi}, n = 1,2,3 \ldots$ **(2)**
    - The n indicates that there can only be certain energy levels
  - $E_1 - E_2 = hv$ **(3)**
    - This is a photon of energy

$$F_C = m_e a = \frac{m_e v^2}{r}$$
$$F_E = \frac{ke^2}{r^2}$$
$$F_C = F_E$$
$$\frac{e^2}{4\pi\varepsilon_0 r} = m_e v^2$$
$$mvr = \frac{nh}{2\pi}$$
$$v = \frac{nh}{2\pi mr}$$
$$\frac{e^2}{4\pi\varepsilon_0 r} = \frac{n^2 h^2 m_e}{r^2 {m_e}^2 (2\pi)^2}$$
$$r = \frac{n^2 h^2 \varepsilon_0}{m_e \pi e^2} = n^2 a_0 \quad \textbf{(4)}$$

    - a_0 is the first Bohr radius
  - $E_n = \frac{-13.6eV}{n^2}$ defines energy for each atomic orbital

1924

- DeBroglie introduces the idea of matter waves

$$E = \gamma mc^2$$

$\gamma \geq 1$ (relativistic parameter)

$$P = \gamma mv$$

$$m = \frac{P}{\gamma v}$$

As v approaches c, γ goes to ∞

$$E = \frac{\gamma Pc^2}{\gamma c}$$

$$E = Pc$$

$$P_{photon} = \frac{E}{c}$$

$$E = hv$$

$$v = \frac{c}{\lambda}$$

$$P = \frac{hv}{c} = \frac{hc}{c\lambda} = \frac{h}{\lambda}$$

$$P_{electron} = \frac{h}{\lambda_e} \qquad \textbf{(5)}$$

1926

- Schrodinger's Equation **(6)**
    - Published by Erwin Schrodinger in 1926, this equation is a linear partial differential equation which describes the wave function of a system, also known as the quantum state.
    - Solutions to this equation can be used to describe many systems; however, in this case, we are using it to describe the discrete energy levels found in the atom.
    - Building on DeBroglie's idea of matter waves, the solution reveals that the frequency of a particle is directly proportional to the total energy **(7)** of a system.

- Infinite Potential Well Example

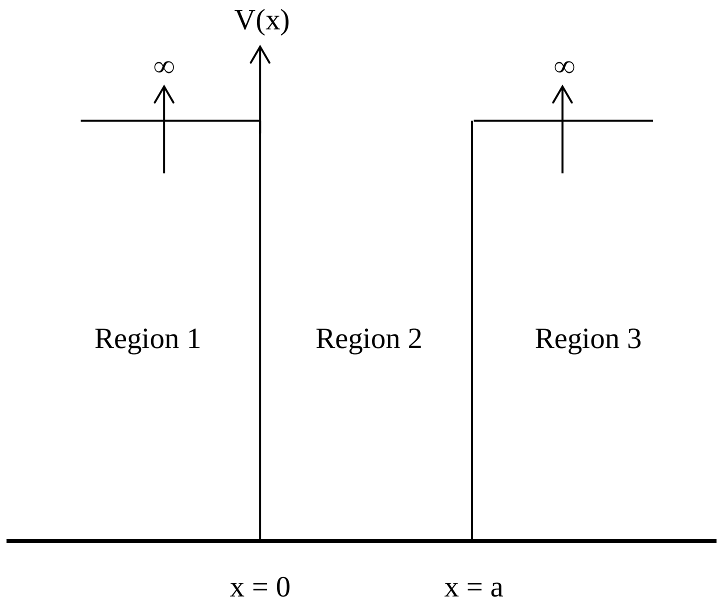


Figure 4: Potential Function of the infinite potential well.

"Semiconductor Physics and Devices," 3rd edition, by Donald A. Neamen, McGraw Hill, 2003.

- This illustration supposes that there is a particle in free space surrounded by impenetrable barriers. As the space becomes narrower, the particle will only be found in certain places since it can only occupy certain energy levels.
- This model is one of the simplest since it can be solved using the mass of the particle and width of the well. It gives insight into quantum mechanics without complex mathematics.
- **Equation 8** is the result of the infinite potential well example using the Schrodinger equation and initial conditions.

$$\frac{-\hbar^2}{2m} \cdot \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = j\hbar \frac{\partial \psi(x,t)}{\partial t} \quad \textbf{(6)}$$

$$\Psi(x,t) = \Psi(x)\phi(t)$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}\big(E - V(x)\big)\psi(x) = 0$$

$$\psi(x) = A_1 \cos(Kx) + A_2 \sin(Kx)$$

$$\psi(0) = A_1 \cos(0) + A_2 \sin(0) = 0$$

$$A_1 = 0$$

$$\psi(a) = A_1 \cos(Ka) + A_2 \sin(Ka) = 0$$

$$Ka = n\pi$$

$$K = \frac{n\pi}{a}$$

$$\int_0^a (A_2 \sin(Kx))^2 dx = 1$$

$$A_2 = \sqrt{\frac{2}{a}}$$

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), n = 1,2,3 \ldots \quad \textbf{(7)}$$

$$K = \sqrt{\frac{2mE}{\hbar^2}}^2 = \left(\frac{n\pi}{a}\right)^2$$

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, n = 1, 2, 3 \ldots \quad \textbf{(8)}$$

Stimulated Emission, Photon Amplification, and Lasers

As demonstrated above, if an electron absorbs a photon of energy, it can be excited from a lower energy state (E_1) to a higher energy state (E_2). Then, when the electrontransfers from its excited state, it emits a photon equal to the change in energy level ($E_2 - E_1 = hv$). This process usually occurs spontaneously, or it can be induced by another photon. This is called *stimulated emission* and is the foundation upon which laser principles are based. When an excited electron is hit with a photon with energy $E_2 - E_1 = hv$, the electron transits from E_2 to E_1. *Population inversion* is when more electrons are at E_2 than E_1; however, three energy states are required for this to occur. Using the ruby laser as an example, electrons are stimulated to the third energy level with photons = $E_3 - E_1 = hv_3$. This is called *pumping*. From E_3, the electrons spontaneously transition to E_2, which is known as a *metastable state* in which the electrons can exist for a relatively long period of time before they transfer to E_1. Therefore, the electrons accumulate at E_2 causing a population inversion. When one electron spontaneously decays to E_1, it causes the other electrons in E_2 to also decay, emitting light as a large collection of coherent photons. This emission is called the *lasing emission*. This entire process describes *photon amplification*, and the device which accomplishes this is a *laser*, an acronym for Light Amplification by Stimulated Emission of Radiation.

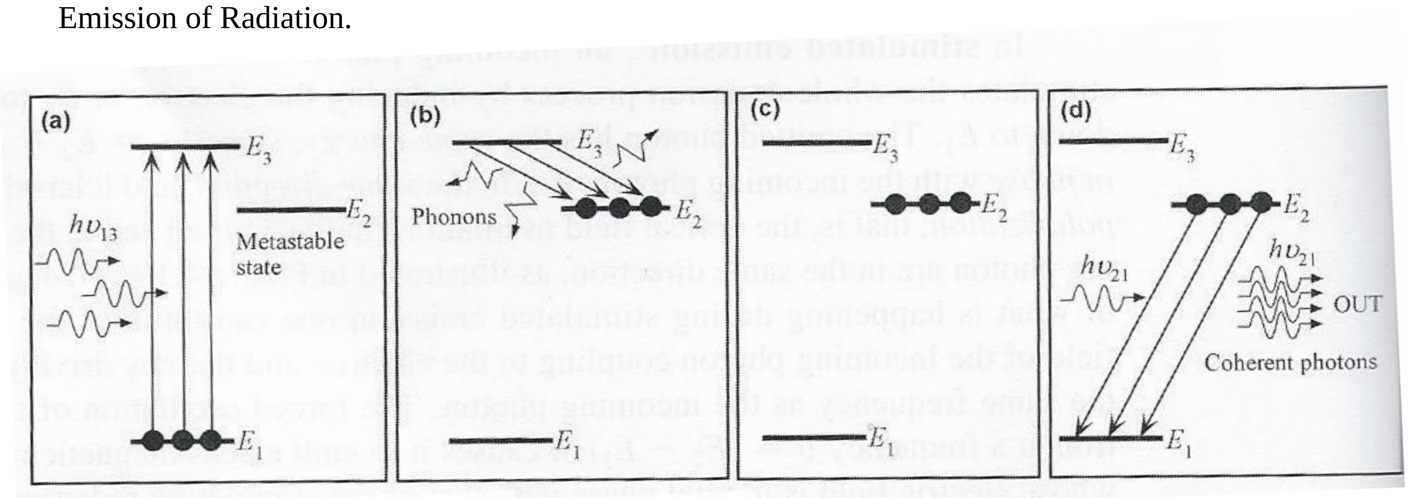


Figure 5: Light Amplification by Stimulated Emission of Radiation

"Optoelectronics and Photonics, Principles and Practices," 2nd edition, by S. O. Kasap, New York, 2013.

Helium-Neon (He-Ne) Laser

The ruby laser is a relatively simple version of a laser; however a Helium-Neon laser, which actually has four energy levels, is used in our research. The helium atoms are excited to a higher energy state which then excites the neon atoms to a metastable state. The electrons are then induced to fall to a lower energy level and emit coherent light. This process emits light at 632.8 nm, giving the He-Ne laser its well-known red color. This is demonstrated in **Figure 6** below.

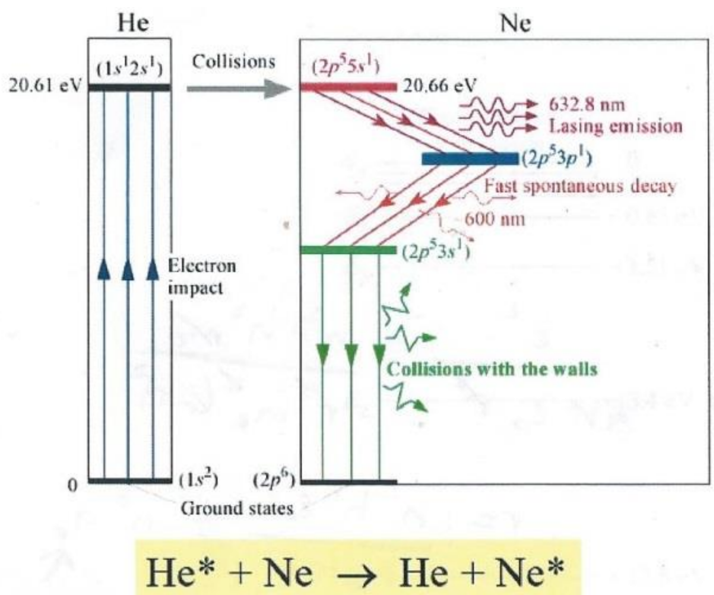


Figure 6: The principles of operation of the He-Ne laser and the He-Ne laser energy levels involved for 632.8 nm emission.

"Optoelectronics and Photonics, Principles and Practices," 2nd edition, by S. O. Kasap, New York, 2013.

While the internal process has been discussed in detail, the actual assembly of the He-Ne laser is very intricate as well. A typical He-Ne laser contains a helium and neon gas mixture within a narrow glass tube and the ends are sealed with a flat mirror on one end and a concave mirror at the other, forming an optical cavity. This emits a *Gaussian beam*.

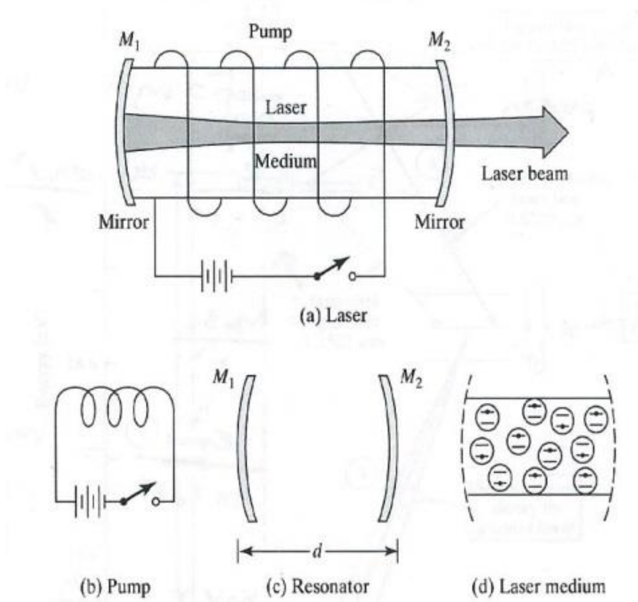


Figure 7: Functionality of a laser

“Introduction to Optics,” 3rd edition, by Pedrotti, Pedrotti and Pedrotti, Pearson, 2007.

Related Paper Analysis

I have read numerous papers about laser beam propagation. Included below are summaries of the ones which focus on what have come to be known as pseudo-partially coherent beams (PPCBs).

Pseudo-Partially Coherent Beam for Free-Space Laser Communication

David Voelz and Kevin Fitzhenry

This paper describes a process to "modulate or temporally alter the phase front of the beam before transmission" which would reduce scintillation at the target and improve the use of a laser for communication purposes. The research seeks to minimize scintillation and beam spread so that a large receiver is not required to detect the beam. Scintillation arises from atmospheric turbulence which disrupts the laser beam. This paper theorizes that by sending many different beams very quickly through the turbulence, at the receiver the entire beam will be "filled in" per say. The results from this paper discuss that while manipulation of the beam caused greater spread, the results of scintillation were reduced.

This paper directly applies to our research because by using the SLM we can alter the phase of the laser beam creating a different beam which will hopefully propagate through the atmosphere better than the original. Presently, we are exploring the idea of cycling, using 4000 screens produced by MATLAB with different degrees of correlation. Using Far Field Transforms, we can predict the outcome of the beam and are looking into propagating "flat top" beams which are more spread out over a larger region rather than having the standard high intensity point in the center. Therefore, these beams have a higher probability of reaching a certain intensity at the target, which could then be translated into a message of some kind. This paper discusses cycling, which in combination with the flat top beam, could prove very successful at propagating through the atmosphere and resisting its effects.

Numerical Investigation on Propagation Effects of Pseudo-Partially Coherent Gaussian Schell-model Beam in Atmospheric Turbulence

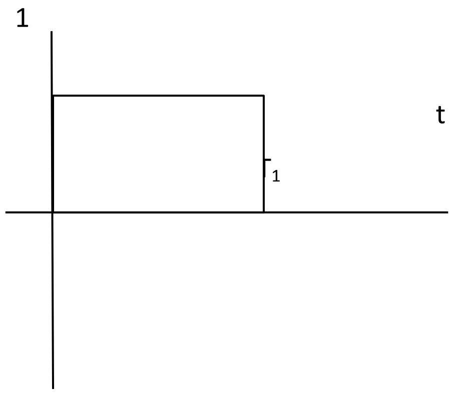
Xianmei Qian, Wenyue Zhu and Ruizhong Rao

Similar to the previous paper, this discusses the effects of Pseudo Partially Coherent Beams (PPCBs) and their advantages and disadvantages over coherent beams. PPCBs use the theory that during the period that the atmosphere has changed once, the phase screen has cycled multiple times. This paper found that the beam radius of a PPCB is always larger than a Coherent Beam (CB) in free space and turbulence. However, the beam radius of a PPCB is the same in turbulence as free space, whereas the CB beam radius is changed based on the atmosphere. Thus, partially coherent beams are less affected by atmospheric turbulence than coherent beams. The paper also reveals that the beam wander of the CB is larger than that of a PPCB; however, the difference is very slight. Interestingly, the scintillation index of a PPCB is lower than that of a CB, which is the goal of our research. Therefore, this suggests that PPCBs are more resistant to the influences of a turbulent atmosphere, such as that found in the maritime environment. Finally, since PPCBs do have larger beam spread than CBs, this would result in a larger dispersion of energy. Thus, there must be a balance between intensity delivered and lower scintillation.

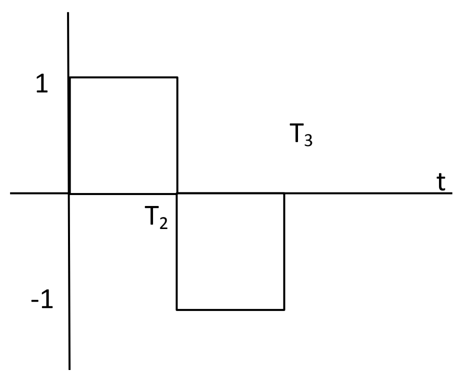
Convolution Project

The design goal of this project is to reduce the amount of noise or the influence of the unwanted signal at the output. Two types of filters are suggested for consideration. The code for this project is referenced in the Appendix.

Low Pass Filter



High Pass Filter



Sinusoidal Input with Noise through Low Pass Filter

Results

Trial	Length of filter, T_{filter}	Power of input signal, S	Power of noise, N	Ratio of Output Signal to Noise, Ratio_out	Power of Output Signal, S_{out}
1	2.50E-04	0.006	0.0744	247.4039	18.3959
2	5.00E-04	0.006	0.0744	537.5714	39.9714
3	7.50E-04	0.006	0.0744	494.0394	36.7346
4	1.00E-03	0.006	0.0744	385.6259	28.6743

From these results, the low pass filter with length 5.00E-04 extracted the input signal the best because it has the highest ratio of output signal to noise. The graphs produced from this convolution are displayed below to visually display what convolution does for a signal and noise. It can be seen that the filter with length 5.00E-04 produces the graph most similar to the input signal.

Output

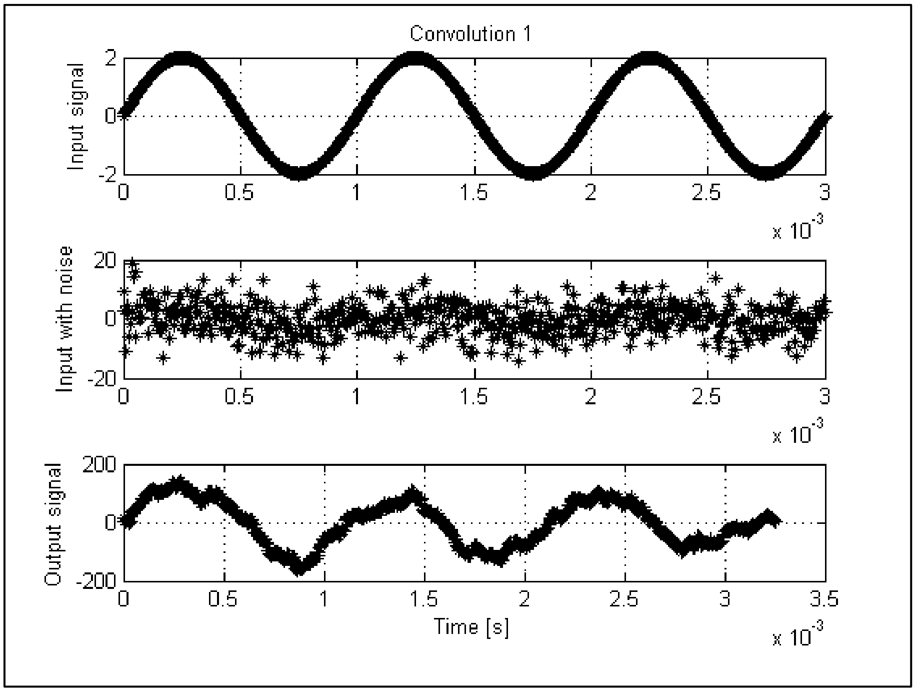


Figure 8: Filter length = 2.50E-04

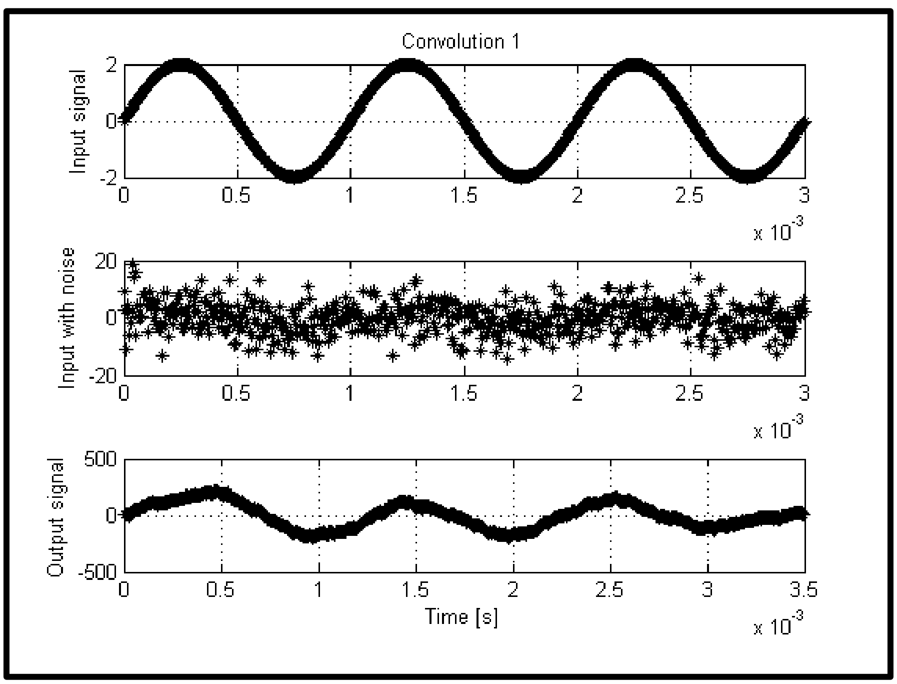


Figure 9: Filter length = 5.00E-04

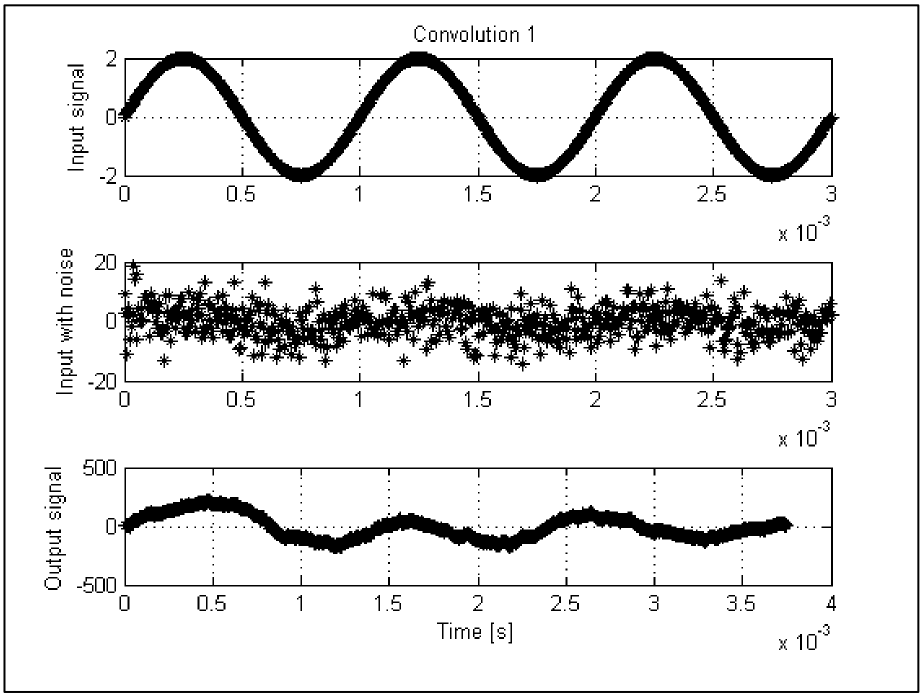


Figure 10: Filter length = 7.50E-04

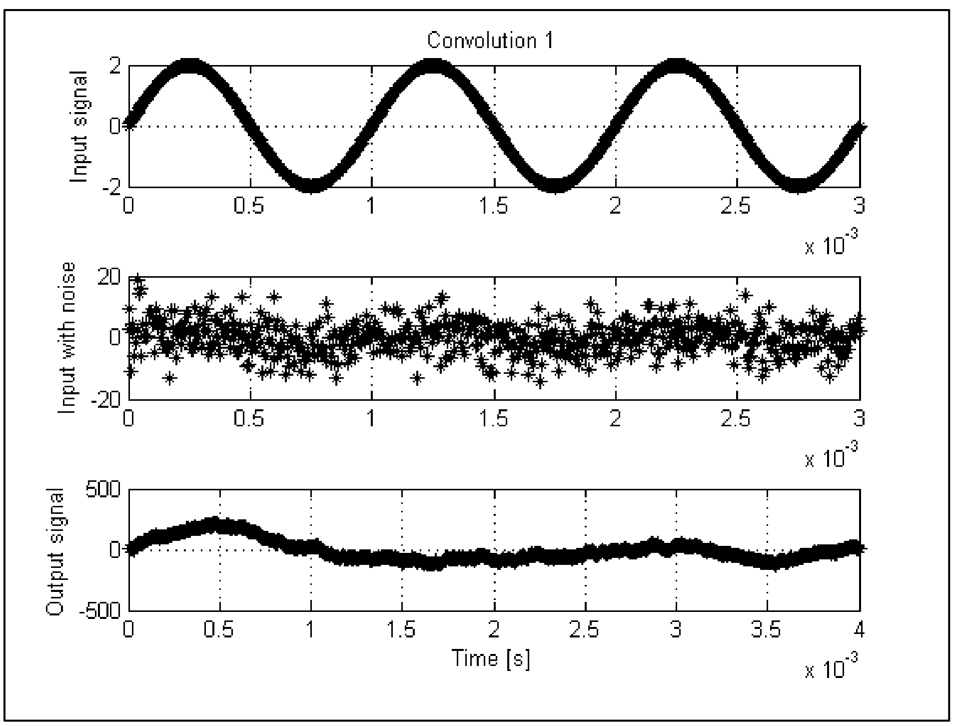


Figure 11: Filter length = 1.00E-03

Two Sinusoidal Inputs through Low Pass Filter

Results

Trial	Length of filter, T_{filter}	Power of signal 1, S_1	Power of signal 2, S_2	Power of Output Signal, S_{out}
1	0.25	1.5	0.375	3.08E+05
2	0.5	1.5	0.375	6.64E+05
3	0.75	1.5	0.375	5.13E+05
4	1	1.5	0.375	3.14E+05

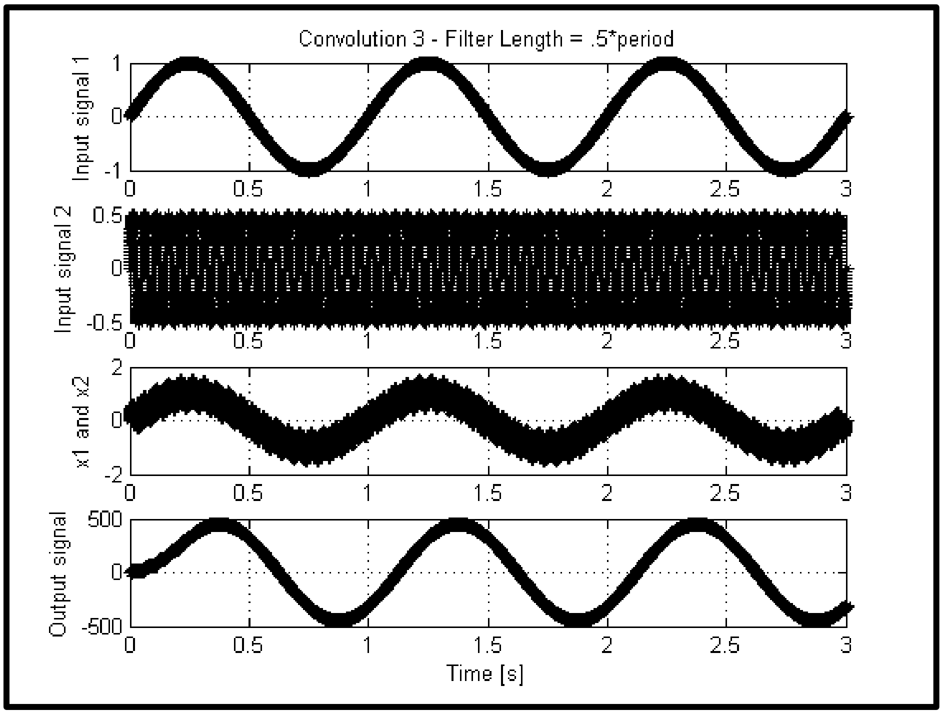


Figure 12: Filter length = .5

In this case, the low pass filter was only effective in extracting the larger sinusoid whereas the goal was to extract the smaller sinusoidal signal.

Sinusoidal Input with Noise through High Pass Filter

Results

Trial	Length of filter, T_{filter}	Power of input signal, S	Power of noise, N	Ratio of Output Signal to Noise, Ratio	Power of Output Signal, S_{out}
1	2.50E-04	0.006	0.0785	193.5239	15.1867
2	5.00E-04	0.006	0.0785	403.9459	31.6994
3	7.50E-04	0.006	0.0785	359.1667	28.1069
4	1.00E-03	0.006	0.0785	290.5121	22.7978

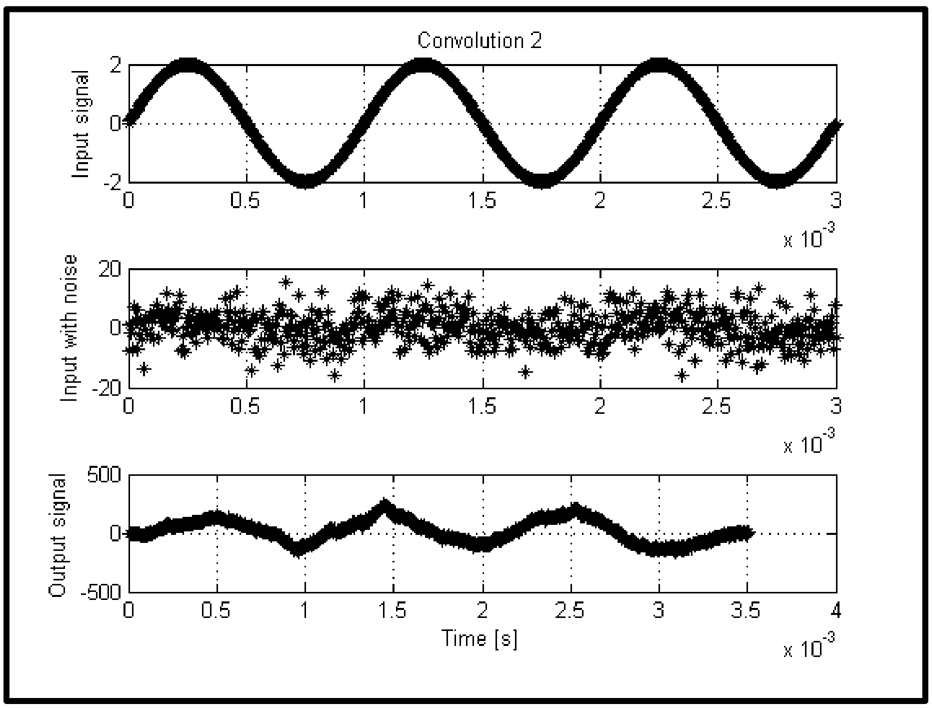


Figure 13: Filter length = 5.00E-04

The high pass filter with length 5.00E-04 was again the best at extracting the input sinusoid.

Two Sinusoidal Inputs through High Pass Filter

Results

Trial	Length of filter, T_{filter}	Power of signal 1, S_1	Power of signal 2, S_2	Power of Output Signal, S_{out}
1	1.00E-04	1.5	0.375	.0015
2	1.00E-03	1.5	0.375	7.5119
3	1.00E-02	1.5	0.375	731.7152
4	1.00E-02	1.5	0.375	5.82e+04

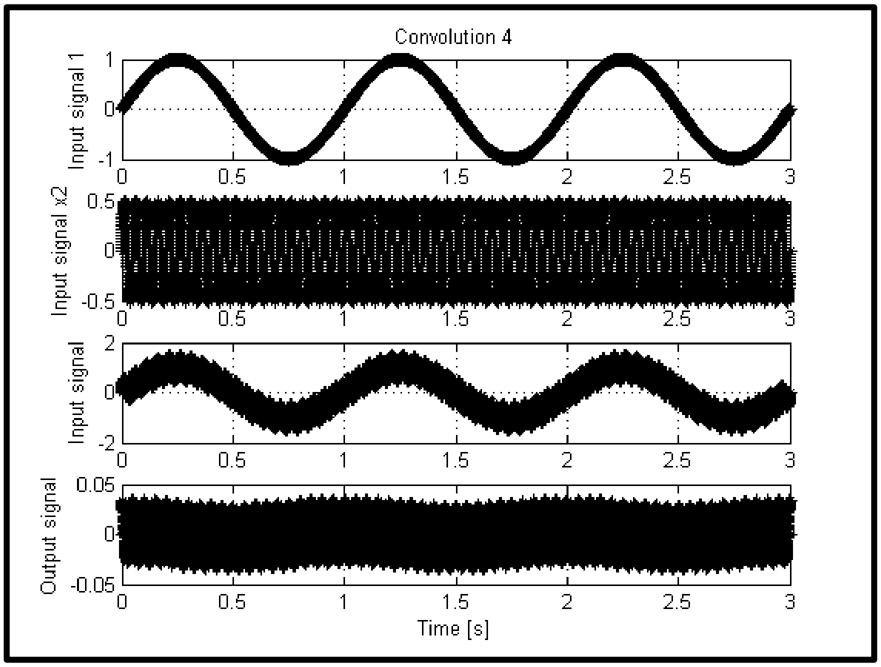


Figure 14: Filter length = 1.00E-04

The high pass filter with length 1.00E-04 had the lowest power of output signal; however, it was the filter that best extracted the smaller sinusoid rather than the larger.

Conclusion

Manipulating the length of the filter in all four cases allowed different signals to be extracted with success. Convolution 1 extracted the clearest sinusoidal signal from the noise while Convolution 4 extracted the smaller sinusoid from the larger one with success. This technique is what we used to modify the phase screens in MATLAB for the Spatial Light Modulator.

Research Summary

Research in developing lasers for use in communications and as a directed energy weapon in the maritime environment is being conducted. However, because of the effects air turbulence, optical turbulence, and temperature have on a beam, using lasers in these areas has not yet been utilized to its full potential. Turbulence can result in increased scintillation which causes fades at the target. By manipulating the phase of the beam using spatial light modulation, a new beam can be created with a “flat top” profile rather than the typical Gaussian form. Voelz and Fitzhenry have discussed a similar beam, formally known as a pseudo-partially coherent beam (PPCB). This research propagates PPCBs through a hot air turbulence emulator capable of simulating low to strong fluctuation conditions, recording the intensity and scintillation at the target. The PPCBs can be created with different spatial coherence levels, which can be compared to the standard Gaussian form, to determine their success in propagating through the turbulence. In addition, this research explores the effects that cycling different phase screens has on the success of propagating the PPCBs through turbulence. Theory predicts that by cycling PPCBs at a rate faster than the atmosphere changes will result in decreased intensity fluctuations at the target. The goal of this project is to minimize scintillation variations at the target after passing the beam through high turbulence.

This past semester, we succeeded in setting up the turbulence emulator, constructed by Assistant Professor Nelson for his dissertation. Also, we propagated laser beams through the emulator using phase screens constructed using MATLAB and convolution theory. These phase screens had different coherence levels, with black corresponding to perfect phase coherence. Finally, we succeeded in running trials to obtain preliminary results on the PPCBs described by Voelz and Fitzhenry.

Equipment

- *ThorLabs HNL020L 632.8 nm Helium Neon Laser*
- *BNS Spatial Light Modulator (SLM) – XY Series*
- *ThorLabs DCU224M - CCD Camera*
- *11 Amp Variable Temperature Heat Gun*
- *Thorlabs BEDS-10-A Expander*
- *Thermaltake Fans*
- *Omega 12 Channel Temperature Recorder RDXL 12SD*

Emulator Setup

An emulator is used to mimic some of the effects of high turbulence found by propagating a beam over a long range in just a laboratory setting. It provides quick and easy control over turbulence strength, is statistically repeatable, provides a random optical turbulence as compared with a static phase screen that is rotated and has a repeating phase pattern, and is modular and extendable. The setup includes four heat guns providing thermal flow of 200 °F which are opposed by four fans providing ambient air counter flow. The heat from the guns is dispersed by three diffuser screens set in front of the heat guns. There are temperature probes spaced evenly throughout taking temperature readings every second. Previous analysis of these temperature changes categorized the turbulence as approximately Kolmogorov along the beam propagation axis with an average C_n^2 value of 3.81E-11. **Figures 15** and **16** below illustrate the setup.

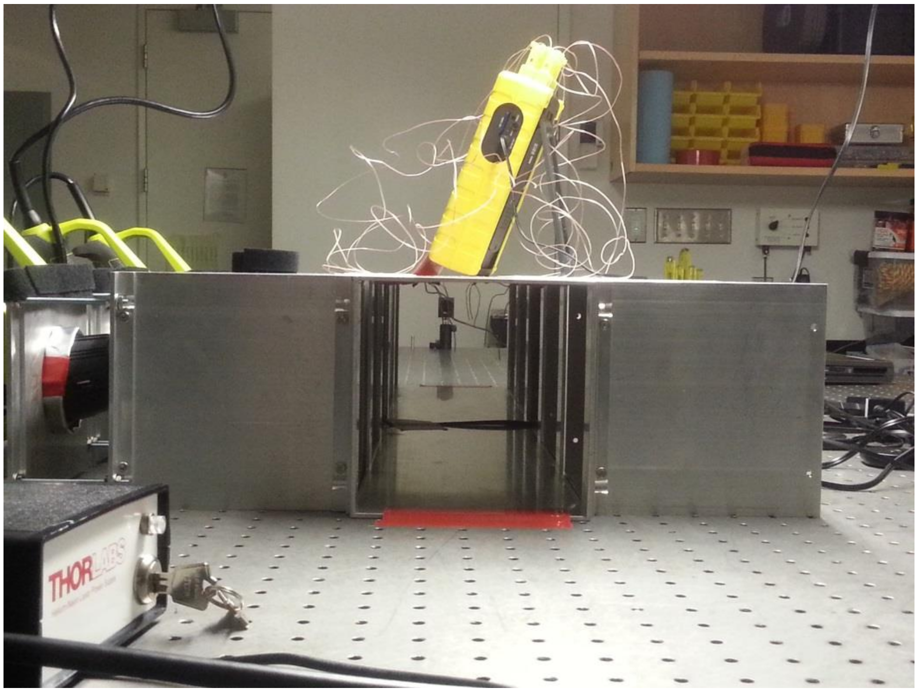


Figure 15: Laser's path through emulator into DC camera

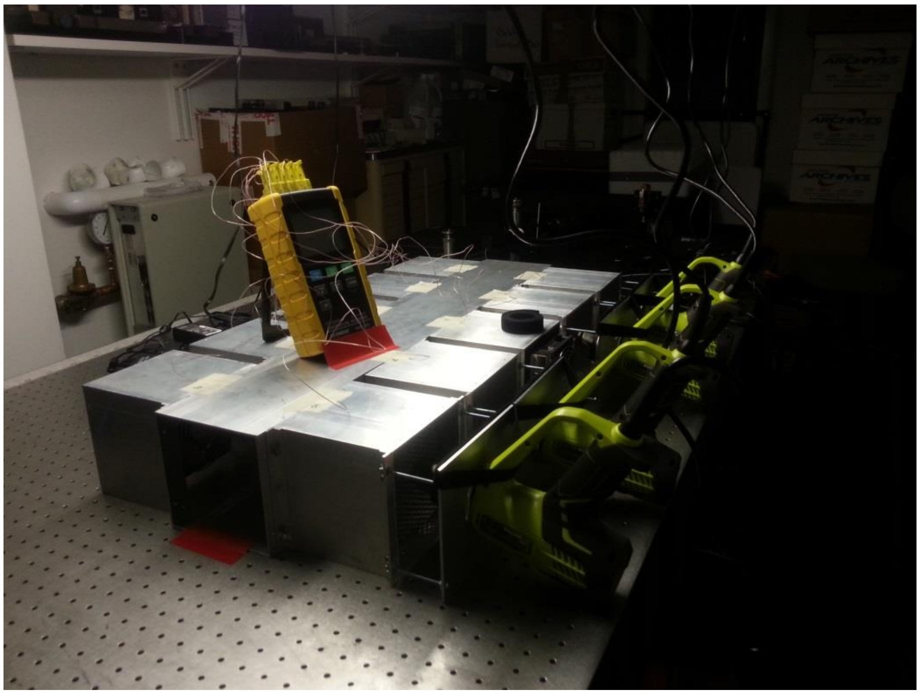


Figure 16: Side view of emulator displaying heat guns and Temperature Recorder

Laser, Expander, and SLM Setup

The laser is first propagated through an expander and then reflected off of the SLM, as shown in **Figure 17**.

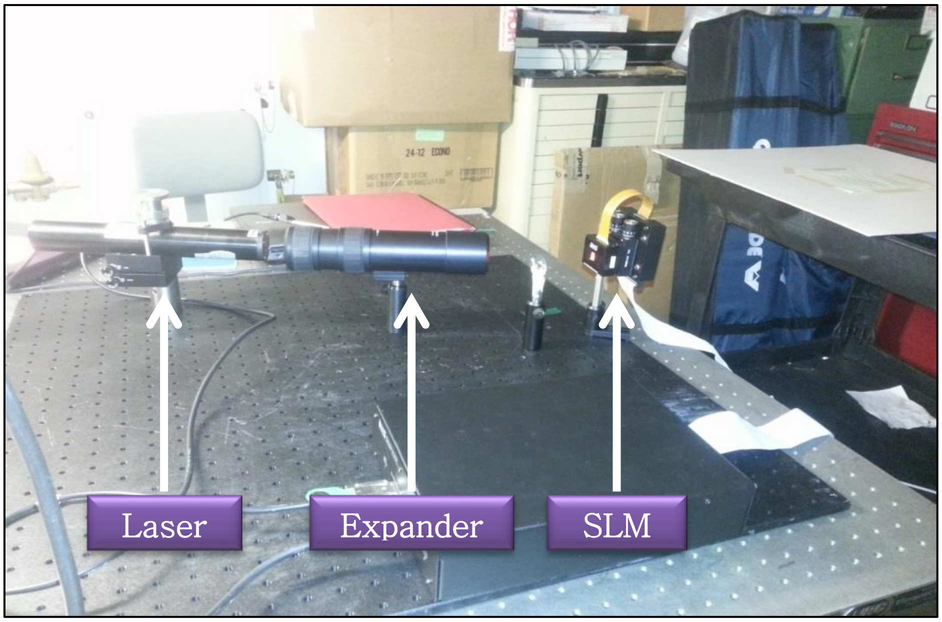
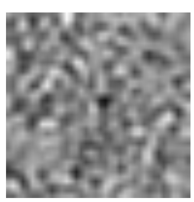


Figure 17: Laser, Expander and SLM setup

SLM Screens

Using MATLAB and convolution, the screens for the SLM can be produced. The Multi-Gaussian Schell Model is used to make the screens, using different levels of coherence. Some sample screens are shown below.

Black
(perfectly coherent)



SLM Screen 2
(strong diffuser)

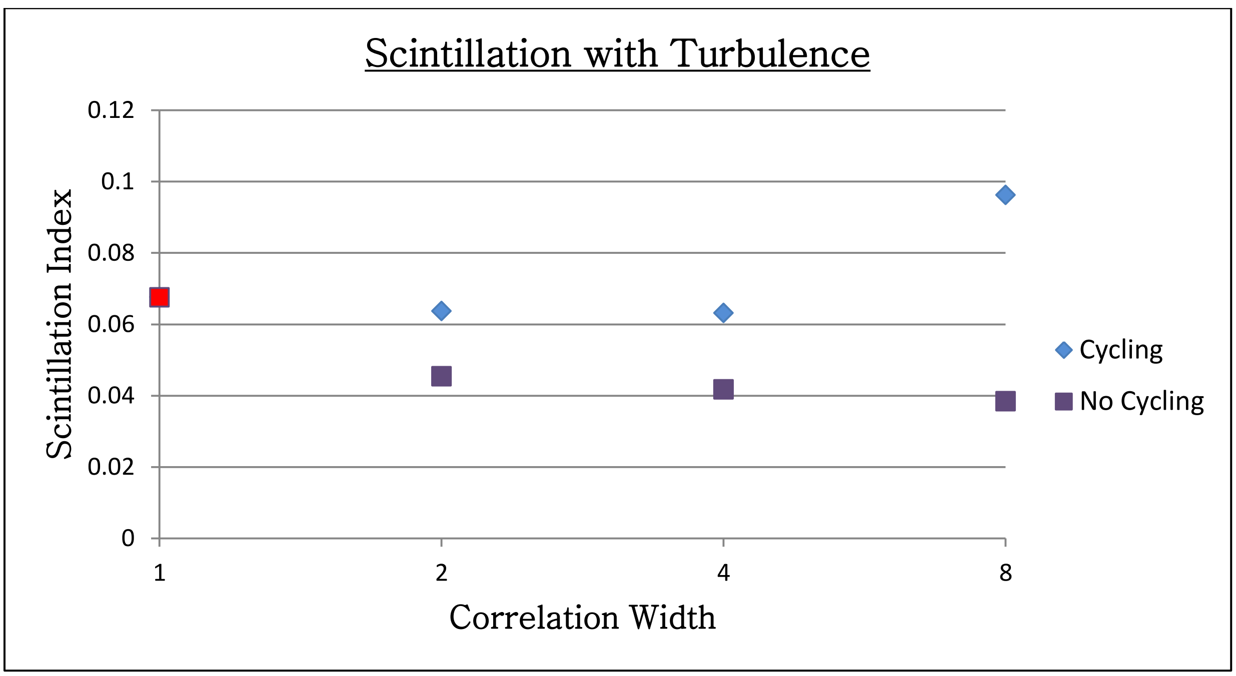
SLM Screen 8
(weaker diffuser)

Process

- Start camera and SLM
- Warm up heat guns and fans
- Begin temperature collection
- Begin cycling
- Begin camera recording
- Convert video to jpeg images
- Analyze jpegs in MATLAB to find scintillation index and intensity

Results

Below is a graph of scintillation index versus coherence. When propagating beams, scintillation should be minimized. The data indicates that all but one of the partially coherent beams had a lower scintillation index than that of black.



Conclusion

This experiment's goal was to examine the effects of pseudo-partially spatially coherent beams through turbulence. Ideally, the scintillation index would be minimized. However, initial results indicate that the scintillation was lower for all of the partially coherent beams. In addition, the pseudo-partially coherent beams had higher scintillation than the partially coherent beams through turbulence. This may be due to the fact that the SLM affected the pseudo-partially coherent beams when changing phase screens so rapidly. Further study will allow us to explore how to reduce scintillation using pseudo-partially coherent beams while maintaining intensity.

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Appendix

Convolution 1: Sinusoidal Input with Noise through Low Pass Filter

Input signals

```
clear
dt = 5*10^-6;
period=1/1000; % Since the signal frequency is 1 kHz
Tx = 3*period;
t=0:dt:Tx; % Let us show 3 periods
x = 2*sin(2000*pi*t);% Input signal
n = 5*randn(size(x));% Generates random noise
S = sum((x.^2)*dt);% Power of the input
N = sum((n.^2)*dt);% Power of the noise
Ratio = S/N;% Ratio between power of input and noise
y = x + n;% Input plus noise
```

Building the filter

```
for i=1:4;

    tfilter = i*.25*period; % Filter lasts .5 signal periods. Modify for best results
    tfup=0:dt:tfilter;
    F=ones(size(tfup));% Builds filter
    c=conv(y,F);% Convolution of filter and input signal with noise
    tconv=0:dt:(length(c)-1)*dt;
    Sout(i) = sum((c.^2)*dt);% Power of the output signal
    Ratio_out = Sout/N;
```

Plotting results

```
    figure(1)
    subplot(311);plot(t,x,'k*');grid;
    title('Convolution 1');
    ylabel('Input signal');
    subplot(312);plot(t,y,'k*');grid;
    ylabel('Input with noise');
    subplot(313);plot(tconv,c,'k*');grid;
    ylabel('Output signal')
    xlabel('Time [s]')

end
```

Convolution 3: Two Sinusoidal Inputs through Low Pass Filter

Input signals

```
clear
dt = .5*10^-3;% Sampling time
period=1;
Tx = 3*period;
t=0:dt:Tx; % Shows 3 periods
x1 = sin(2*pi*t);%
x2 = .5*sin(40*pi*t);
S1 = sum((x1.^2)*dt);% Power from signal 1
S2 = sum((x2.^2)*dt);% Power from signal 2
y = x1 + x2;% Both signals added
```

Building the filter

```
for i=1:4;
```

```
    tfilter = i*.25*period;% Filter lasts 2 signal periods. Modify for best results
    tfup=0:dt:tfilter;
    F=ones(size(tfup));% Builds filter
    c=conv(y,F);% Convolution of filter and input signal
    tconv=0:dt:(length(c)-1)*dt;
    Sout(i) = sum((c.^2)*dt);% Power of output signal
```

Plotting results

```
    figure(i)
    subplot(411);plot(t,x1,'k*');grid;
    title('Convolution 3');
    ylabel('Input signal 1');
    subplot(412);plot(t,x2,'k*');grid;
    ylabel('Input signal 2');
    subplot(413);plot(t,y,'k*');grid;
    ylabel('x1 and x2');
    subplot(414);plot(tconv,c,'k*');grid;axis([0 3 -500 500]);
    ylabel('Output signal')
    xlabel('Time [s]')
```

```
end
```

Convolution 2: Sinusoidal Input with Noise through High Pass Filter

Input signals

```
clear
dt = 5*10^-6;
period=1/1000; % Since the signal frequency is 1 kHz
Tx = 3*period;
t=0:dt:Tx; % Let us show 3 periods
x = 2*sin(2000*pi*t);% Input signal
n = 5*randn(size(x));% Generates random noise
S = sum((x.^2)*dt);% Power of the input
N = sum((n.^2)*dt);% Power of the noise
Ratio = S/N;% Ratio between power of input and noise
y = x + n;% Input plus noise
```

Building the filter

```
for i=1:4;
```

```
    T2 = i*.25*period;
    T3 = i*.25*period;% Filter lasts 2 periods. Modify for best results
    tup=0:dt:T2;% Length of time filter equals 1
    tdown=T2+dt:dt:T3+dt;% Length of time filter equals -1
    FilterHP = [ ones(size(tup)) -ones(size(tdown))];% Builds filter
    c=conv(y,FilterHP);% Convolution of filter with input noise
    tconv=0:dt:(length(c)-1)*dt;
    Sout(i) = sum((c.^2)*dt);% Power of output signal
    Ratio_out = Sout/N;
```

Plotting results

```
    figure(i)
    subplot(311);plot(t,x,'k*');grid;
    title('Convolution 2');
    ylabel('Input signal');
    subplot(312);plot(t,y,'k*');grid;
    ylabel('Input with noise');
    subplot(313);plot(tconv,c,'k*');grid;
    ylabel('Output signal')
    xlabel('Time [s]')
```

```
end
```

Convolution 4: Two Sinusoidal Inputs through High Pass Filter

Input signals

```
clear
dt = .5*10^-3;% Sampling time
period=1;
Tx = 3*period;
t=0:dt:Tx; % Shows 3 periods
x1 = sin(2*pi*t);%
x2 = .5*sin(40*pi*t);
S1 = sum((x1.^2)*dt);% Power from signal 1
S2 = sum((x2.^2)*dt);% Power from signal 2
y = x1 + x2;% Both signals added
```

Building the filter

```
    T2 = .0001*period;
    T3 = .0001*period;% Modify for best results
    tup=0:dt:T2;% Length of time filter equals 1
    tdown=T2+dt:dt:T3+dt;% Length of time filter equals -1
    FilterHP = [ ones(size(tup)) -ones(size(tdown))];% Builds filter
    c=conv(y,FilterHP);% Convolution of filter with input noise
    tconv=0:dt:(length(c)-1)*dt;
    Sout = sum((c.^2)*dt);% Power of output signal
```

Plotting results

```
    figure(1)
    subplot(411);plot(t,x1,'k*');grid;
    title('Convolution 4');
    ylabel('Input signal 1');
    subplot(412);plot(t,x2,'k*');grid;
    ylabel('Input signal x2');
    subplot(413);plot(t,y,'k*');grid;
    ylabel('Input signal');
    subplot(414);plot(tconv,c,'k*');grid;axis([0 3 -.05 .05]);
    ylabel('Output signal')
    xlabel('Time [s]')
```