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Probability Density Function (PDF) of intensity of a stochastic light beam propagating in the turbulent atmosphere

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Abstract. Measurements of the intensity of a light beam produced by a laser source, reflected from a Spatial Light Modulator (SLM) and propagating in a weakly turbulent maritime atmosphere have been carried out in a recent campaign on the grounds of the US Naval Academy. The effect of the degree of spatial coherence in the beam at the source plane (right after the reflection from the SLM) on the Probability Density Function (PDF) of the intensity of the propagating beam at a single fixed position in space is studied in detail. The measured intensity histogram of the fluctuating intensity is compared with the Gamma-Laguerre analytical model for the intensity PDF.

1. Introduction.

Optical waves interacting with the turbulent atmosphere, either in the absence or in the presence of scattering particles, exhibit a number of effects that prevent communication and sensing systems from efficient transfer of information [1, 2]. In order to deal with this problem a number of approaches have been recently suggested including, for instance, spatially sparse receivers, source wavelength and polarization diversity, source partial coherence [3]. Among these mitigation techniques the latter was paid very close attention, however primarily in the theoretical domain [4]. As early as in 1984 it was demonstrated by V. Banakh [5] that the scintillation in an optical beam produced by a partially coherent source is lower than that in a comparable coherent beam. Even though the average intensity and the intensity scintillation may be considered as the major beam characteristics affecting the information transfer, for complete evaluation of the system performance it is necessary to possess the knowledge of the Probability Density Function of the fluctuating intensity of the beam at the detector [2]. To our knowledge, there are neither analytical nor experimental reports in the literature addressing the subject of the PDF of partially coherent beams in turbulent atmosphere.

The purpose of this study is, on the basis of the obtained experimental data, to infer qualitative and quantitative information about the dependence of the shape of the (intensity) probability density function of the beam propagating in the turbulent atmosphere on the source degree of coherence. For efficient generation of partially coherent sources with various coherence widths we employed the nematic, phase-only Spatial Light Modulator. Before and after propagation through the turbulent channel the beam is captured by fast CCD detectors and the intensity histograms are constructed and compared with the mathematical PDF model suggested by R. Barakat some years ago [6]. The paper is organized as follows: in Sec. 2 the review of the partially coherent optical fields and their generation by a phase-only SLM is provided; Sec. 3 concerns with instrumentation and details of data collection; Sec. 4 deals with the digital data processing scheme; Sec. 5 gives the histogram and analytical PDF results.

2. Generation of partially coherent beams

A partially coherent (also called random, stochastic) beam can be generated in a number of ways, for instance, on propagation of a field generated by an incoherent source at a distance or by transmission of a laser beam through a rotating ground glass plate (see [7] for the overview of the techniques, see also [8]). We will employ a fairly new method which is based on the reflection of a laser beam from the phase-only, nematic Spatial Light Modulator (SLM). The surface of the SLM device acts as a phase modulator (phase screen): the resulting phase can be made different for different pixels within one frame. The SLM is capable of rapid change of frames. Regardless of the discrete nature of such phase modulation both spatially and temporally, the resulting field can be regarded as partially coherent provided the detector rate is much slower. The other convenient feature of the SLM is the capability of digitally changing and adjusting the statistics of phase. The operational principle of an SLM is based on the fact that the electric signal (induced voltage) given to a pixel is proportional to the magnitude of the phase modulation of the reflected wave. The assignment of the signals (phases) at all the pixels on the SLM can be done via a simple computer program which returns a 2D array of phase values. In general both deterministic and random phase arrays can be produced. In the case of random case distribution any statistical properties can be prescribed. In this study we will limit ourselves to a Gaussian random process with zero mean and Gaussian second-order correlation function. We note that temporally the sequence of images is uncorrelated for any of the pixels and, hence, can be used as an ensemble of realizations. More specifically, we assume that for each position on the SLM surface with position vector $\mathbf{p} = (x, y)$ the real random process $\phi(\mathbf{p})$ describing the phase distribution on the SLM (called below “SLM phase”) is correlated as

$$\langle \phi(\mathbf{p}_1)\phi(\mathbf{p}_2) \rangle = \phi_0^2 \exp \left[-\frac{(\mathbf{p}_1 - \mathbf{p}_2)^2}{2\delta_\phi^2} \right], \quad (2.1)$$

where $\phi_0^2 = \sqrt{\langle |\phi(\mathbf{p})|^2 \rangle}$, δ_ϕ^2 is the correlation width, and the angular brackets stand for the ensemble of realizations of the SLM phase distributions. In order to simulate such a process we first generate a 2D array of independent random variables $R_\phi(\mathbf{p})$ which obey Gaussian statistics with zero mean. It is well known that the independent Gaussian variables are necessarily uncorrelated i.e.

$$\langle R_\phi(\mathbf{p}_1)R_\phi(\mathbf{p}_2) \rangle = \delta^{(2)}(\mathbf{p}_1 - \mathbf{p}_2), \quad (2.2)$$

where $\delta^{(2)}$ is a 2D delta-function. Next, we convolve array $R_\phi(\mathbf{p})$ with the Gaussian function

$$f_\phi(\mathbf{p}) = \exp\left[-\mathbf{p}^2/\gamma_\phi^2\right] \quad (2.3)$$

to obtain the Gaussian-correlated array $g_\phi(\mathbf{p})$, i.e.

$$g_\phi(\mathbf{p}) = \int f_\phi(\mathbf{p}-\mathbf{p}')R_\phi(\mathbf{p}')d^2\mathbf{p}'. \quad (2.4)$$

Indeed, after integrating we obtain

$$\langle g_\phi(\mathbf{p}_1)g_\phi(\mathbf{p}_2) \rangle = \frac{\pi\gamma_\phi^2}{2} \exp\left[-\frac{(\mathbf{p}_1 - \mathbf{p}_2)^2}{2\gamma_\phi^2}\right]. \quad (2.5)$$

Finally, on associating $\phi_0 = \sqrt{\pi\gamma_\phi/2}$, $\delta_\phi = \gamma_\phi$ we arrive at Eq. (2.1). By repeating the procedure the necessary sequence of the SLM phase images is produced. The following MatLab code can be employed for the SLM phase screen generation.

```
clear all
jj = 1;
for n=1:3000
I = normrnd(0,1,512,512);
I_min = min(min(I));I_max = max(max(I));
[y,x] = meshgrid(0:length(I),0:length(I));
r = length(x)/2; c = length(x)/2; rho = sqrt((x-r).^2 + (y-c).^2);
Corr_width_2 = 1; window = exp(-rho.^2/Corr_width_2);
GSB = conv2(window,I); figure(2)
GSB_512 = GSB_512./max(max(GSB_512)); imshow(GSB_512,[]);
jj = jj+1;
figname = printf('GSM_%d_corr_width_%d_Gaussian.bmp',jj,round(Corr_width_2));
imwrite(GSB_512,figname,'bmp');
end
```

Figure 1 illustrates several phase screens simulated by the procedure described above. On choosing \mathcal{N}_ϕ^2 to take values, say, (A) 1, (B) 100, (C) 300 we find that the corresponding source correlation widths are 0.015mm, 0.15mm, 0.26mm respectively. For example, the SLM window is 7.68 x 7.68 mm and 512 x 512 pixels, hence the correlation width computation for a value 300 is as follows:

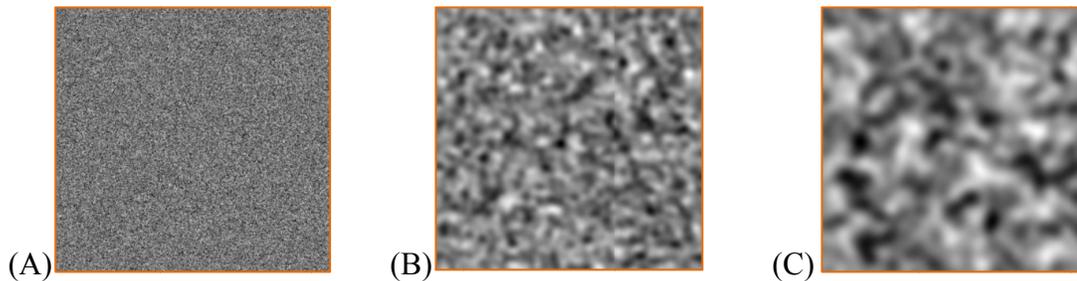


Fig.1 Phase screens generated for the SLM.

3. Experiment

The optical setup used in the experiments for generation of partially coherent waves is shown in Fig. 2. The beam is generated by the a red HeNe laser and passed through a beam expander to achieve acceptable beam diameter for long propagation link. Expanded laser beam is reflected from the Special Light Modulator (SLM) surface. The size of the SLM used in the experiment is 7.68 x 7.68 mm, well aligned with the size of the laser beam coming out of the beam expander. There are 512x512 active pixels for which the index of refraction is controllable, producing 256 different phase levels at the rate of 300 frames per second. Due to the limitations of computing power we modulated the propagation for 19 ms. We put this sequence in the loop to accomplish three minutes of propagation testing.

In order to capture the statistics of the beam at the source, a beam splitter redirected portion of the beam to be recorded using a camera. The intensity of the laser beam on the sensor surface was attenuated using neutral filters and the background light was reduced using a red notch filter After this process the beam captured directly on the ccd sensor. The camera has the sensor with the pixel size of 4.65 x 4.65 μm and spacial resolution of 1280 x 1024 pixels. The camera captured the video of the modulated laser beam and the video file was converted to the sequence of frames with the rate of 10 frames per second and amplitude resolution of 256 different intensity levels. note to myself: Cameras maximum capability is 45 frames per second.

On the target we had a camera with spatial resolution of 128 x128 pixels size of 3.63 x 3.63 μm . The dynamic range allows 4096 different values of light to be recorded on a frame. Due to computational limitations we recorded the light with approximately 200 frames per second. This camera can record the light realizations at the rate of more than 1000 frames per second allowing us to capture even with high speed wind influence on the laser light, or any other influence or modulation of light. Dedicated computer captured about 40000 frames collected during each experimental run. These data sets were processed and the findings were presented in this paper.

The weather conditions at the target was recorded using weather station that had collected the status of temperature pressure, humidity, and a number of other environmental conditions at the rate of one reading per minute. This information was collected in order to correlate weather conditions with the qualitative analysis of the beam propagation properties.

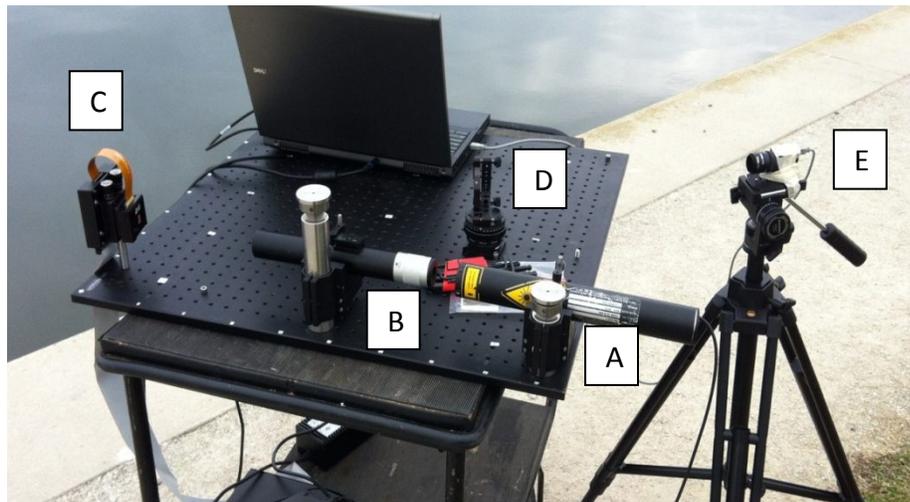


Fig.2 Optical setup for generation of partially coherent beam. A – Laser beam source, B Beam Expander, C – Spatial Light Modulator, D – Beam Splitter, E – Camera,

4. Data post-processing.

All of the data processing is performed in MATLAB. Each frame is imported into MATLAB and matrices I_j are generated in order to get access to the intensity of the light at each pixel. Index j takes values 1 to N , number of frames. Each recorded frame represents one realization of the beam cross-section at the detector plane.

Several tests were performed to see if the beam statistics is different at the different locations on the sensor. It was concluded that the statistics stayed the same; therefore the

processing was simplified by fixing the middle pixel as location at which the intensities for all of the realizations will be collected and vector Q_c was created. The vector Q_c is a time sequence and the random variable that represents the fluctuating light intensity scattered by the atmosphere along the propagation path during recorded time segment. In order to reduce the impact of the background light and capture only the range of changes in the beam intensity, a constant non-zero value lower than the minimum intensity was subtracted from each entry creating the new vector Q_c .

The first moment is the mean M calculated from the vector Q . This value is used to normalize of all other moments which are:

$$\langle I^k \rangle = \frac{1}{N} \sum_{j=1}^N \frac{Q_{c_j}^k}{M^k}. \quad (4.3)$$

The Gamma-Laguerre reconstruction method uses the statistical moments of the recorded light intensities to compute the desired pdf. In order to compare the pdfs with the histogram both intensity levels are normalized by their mean values. Further, the results based on the pdf model are also compared to the recorded fluctuating light intensities histogram using the least square error calculation. The first step in creating a histogram is to find the intensity frequency. To that end a vector D is generated from Q_c using 36 bins. Vector Q_c is searched for $j = 1$ to N to place the intensity of each realization in their respective bins, thus creating the frequency vector D . Each entry of D is the count of intensities associated with its bin number. Vector D is normalized in two steps: first using the mean M and second by making the total area under the histogram curve unity. Once the bin number that contains mean value M is located, its frequency, C , is recorded. The normalized frequency vector T_M is then established:

$$T_M = \frac{D}{C}. \quad (4.4)$$

To account for the unity area under the histogram curve vector, T_M is normalized as follows:

$$N_A = T_M / \sum_{j=1}^{256} T_{M_j} \quad (4.5)$$

Data N_A represents the normalized histogram to be compared with the Gamma-Laguerre pdf model shown in the next section. Although the theoretical model calculates the pdf for the fluctuating light intensity values varying from 0 to infinity, our data N_A contains a finite range of intensities. We will display the pdf computed up to three times the mean value.

The pdf W of the fluctuating intensity, I , gives the probability that the beam's intensity attains a certain level. More precisely, if h is the intensity, normalized by its mean value, i.e. if $h = I/\langle I \rangle$ then

$$\text{Probability}(a < h < b) = \int_a^b W(h) dh . \quad (4.6)$$

The statistical moment of order n is obtained from the intensity's pdf is well known

$$\langle h^n \rangle = \int_0^\infty W(h) h^n dh . \quad (4.7)$$

While calculation of moments from a given pdf is an easy task, the reconstruction of the pdf from the first measured moments is considerably more involved. For problems involving light propagation in random media several pdf reconstruction procedures have been suggested which lead to well-known pdf models. In this work we will only be concerned with one model introduced some years ago by Barakat [6]. This model is only based on the first several statistical moments of the fluctuating intensity, and does not require the knowledge of the atmospheric parameters and propagation distance. Also, it is valid everywhere in the beam cross-section, and it takes into account possible scattering and absorption from particles and aerosols. More importantly, it can deal with any source generating the beam, including the whole variety of partially coherent sources.

The approach in [6], which we will refer to as the Gamma-Laguerre model, suggests finding several first moments of intensity with the help of the Gamma distribution weighted by generalized Laguerre polynomials. More precisely, it has the form

$$W_{GL}(h) = W_g(h) \sum_{n=0}^{\infty} W_n L_n^{(\beta-1)}\left(\frac{\beta h}{\mu}\right), \quad (4.8)$$

where $W_g(h)$ is the Gamma distribution given by

$$W_g(h) = \frac{1}{\Gamma(\beta)} \left(\frac{\beta}{\mu}\right)^\beta h^{\beta-1} \exp\left(-\frac{\beta h}{\mu}\right), \quad (4.9)$$

with Γ being a Gamma-function, and the two parameters of the distribution are defined through the first and second moments as:

$$\mu = \langle h \rangle, \quad \beta = \langle h \rangle^2 / (\langle h^2 \rangle - \langle h \rangle^2). \quad (4.10)$$

Further, W_n are the weighing coefficients:

$$W_n = n! \Gamma(\beta) \sum_{k=0}^n \frac{(-\beta/\mu)^k \langle h^k \rangle}{k!(n-k)! \Gamma(\beta+k)}. \quad (4.11)$$

By definition $W_0 = 1$, $W_1 = 0$ and $W_2 = 0$ since the first two moments define μ and β . The generalized Laguerre polynomials $L_n^{(\beta-1)}(x)$ entering formula (8) are given by the expressions

$$L_n^{(\beta-1)}(x) = \sum_{k=0}^n \binom{n+\beta-1}{n-1} \frac{(-x)^k}{k!}. \quad (4.12)$$

The Gamma-Laguerre model was successfully used by the authors in [9] for reconstruction of the intensity PDF of a field produced by a laser source after it interacts with maritime atmosphere.

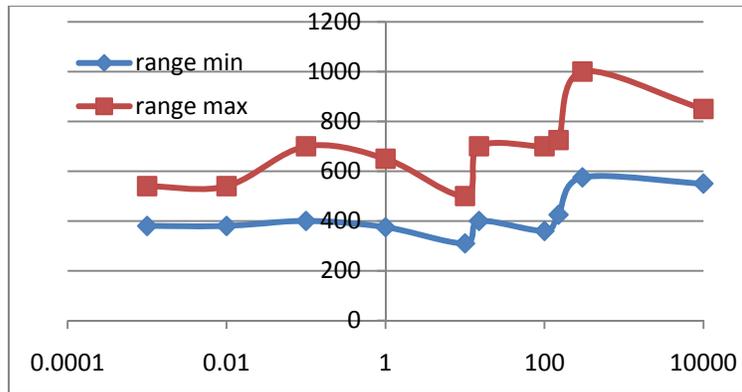
5. Results.

The results of the experiment and post-processing are summarized in Table 1 and in Figs. 3 and 4. Table 1 presents the 10 measurements carried out for different SLM cases, i.e. for different typical SLM phase correlations. The data include the minimum (MIN) and the maximum (MAX) values of the range, the maximum of the PDF (PEAK) curve and the scintillation index (SI).

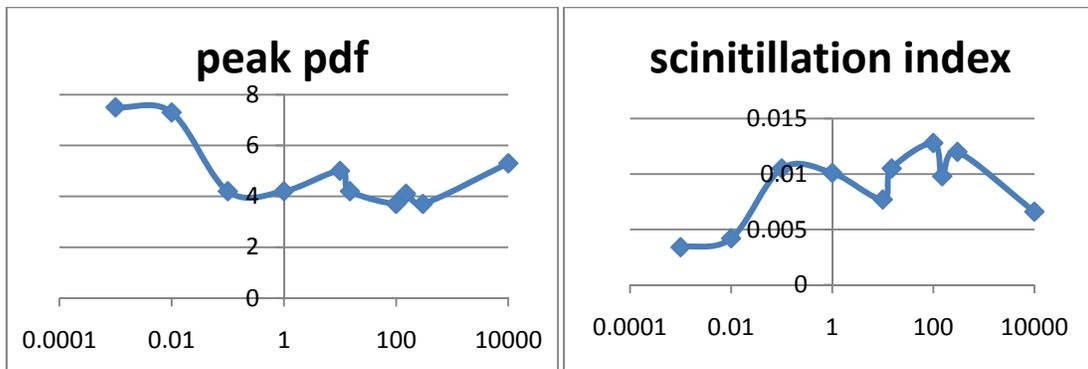
Table 1. Results of the experiment for SLM phase correlations.

SLM case	MIN	MAX	PEAK	SI
0.001	380	540	7.5	0.0034
0.01	380	540	7.3	0.0042
0.1	400	700	4.2	0.0105
1	375	650	4.2	0.0101
10	310	500	5	0.0077
15	400	700	4.2	0.0105
100	360	700	3.7	0.0128
150	425	725	4.1	0.0098
300	575	1000	3.7	0.012
No SLM	550	850	5.3	0.0066

Figure 3 graphically represents the entries of Table 1 with (A) the minimum and the maximum of the intensity ranges, (B) the PDF peak values and (C) the values of the scintillation index, as a function of SLM case. Figure 4 represents the actual data for each run in the form of histograms and fitted Gamma-Laguerre PDF curves (where possible). Namely, in the left column the histograms of the intensity measured by the ccd camera at a distance of 150 m from the source are given while on the right the normalized bins and PDFs where possible are provided. The Scintillation Index [SI] values are also calculated for each case and provide a rough estimate of the width of the distribution. Note that $SI=1/\beta$, where β is given in Eq. (4.10).



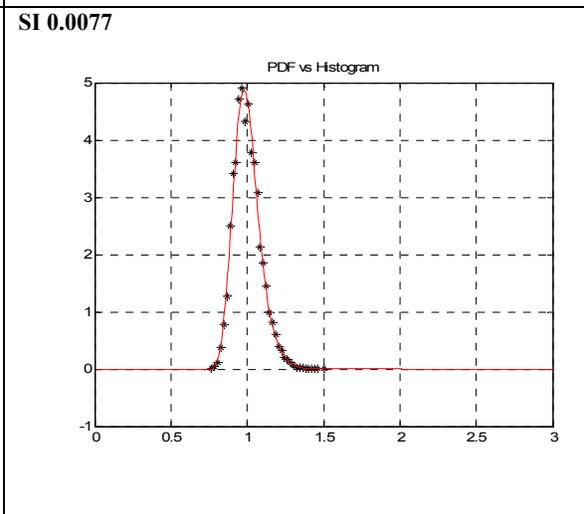
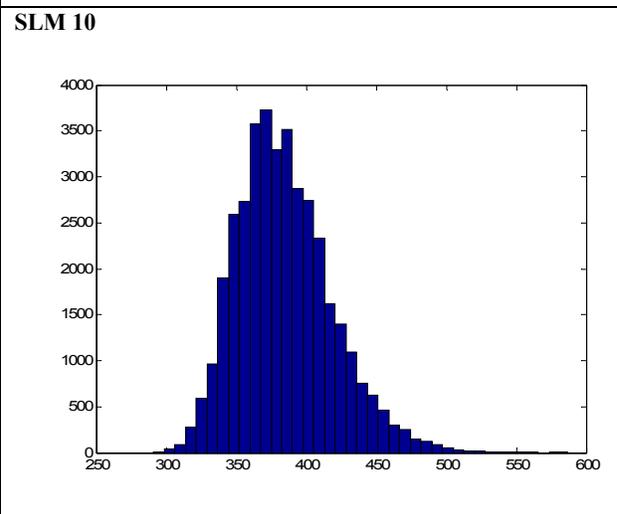
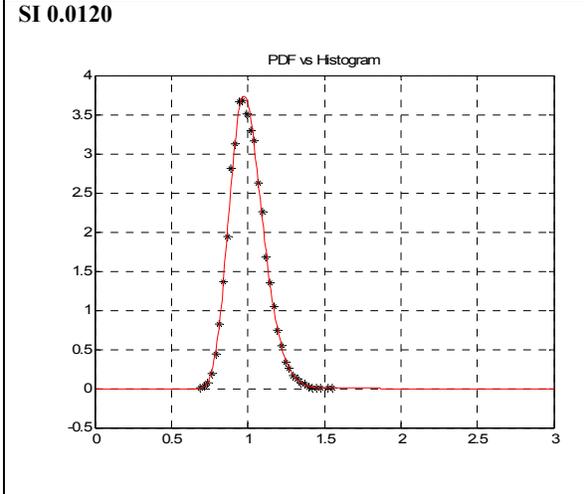
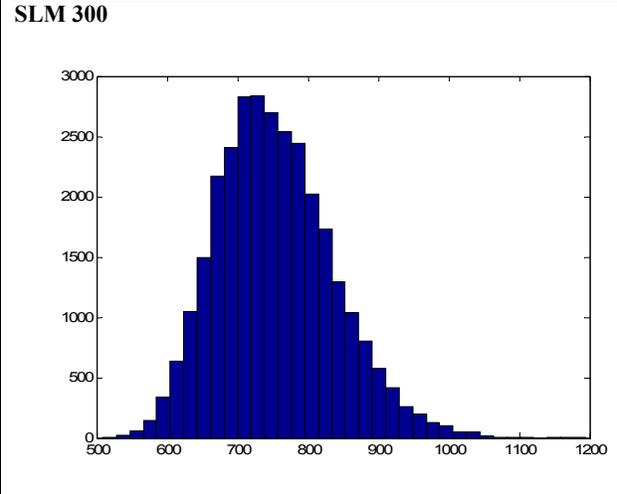
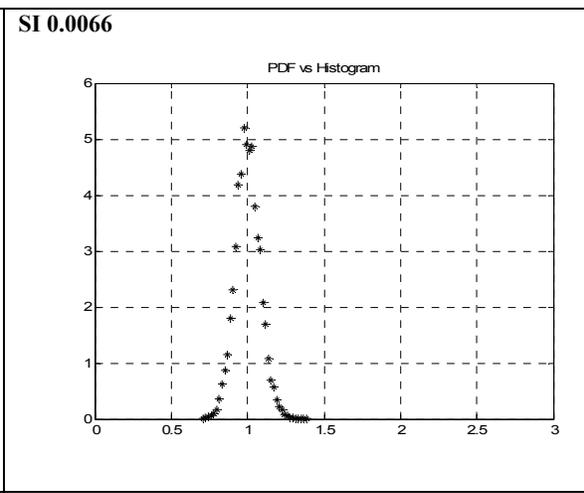
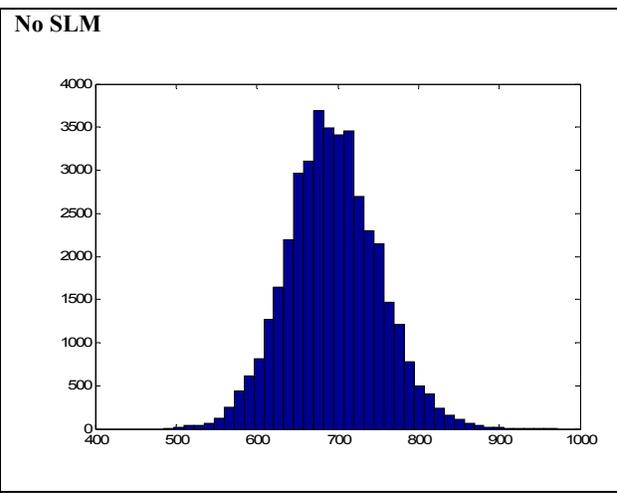
(A)



(B)

(C)

Fig. 3 Summary of some data statistics for various source phase correlations.



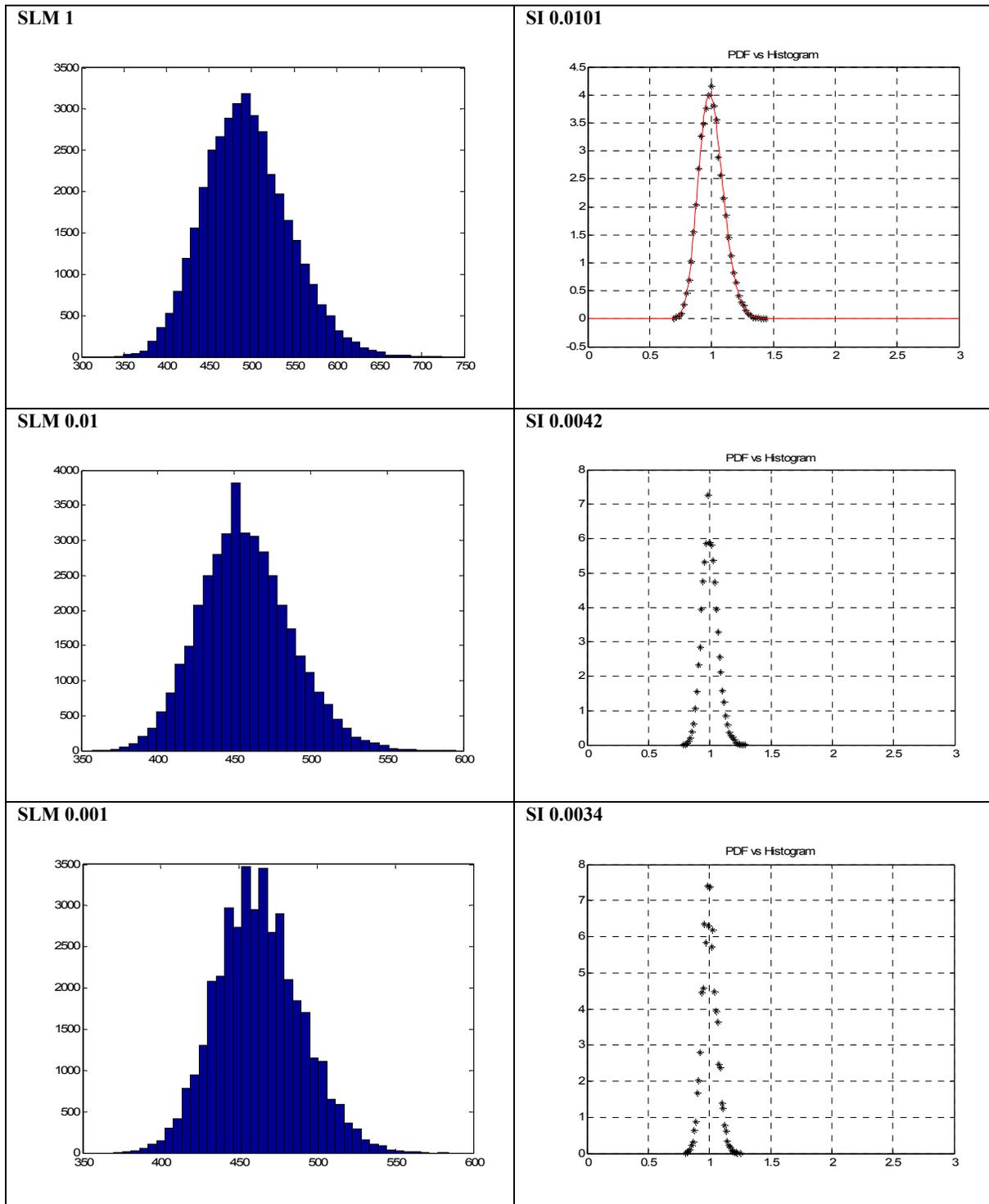


Fig. 4 The histograms of measured intensity and the PDFs of normalized intensity.

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