

# PROCEEDINGS OF SPIE

[SPIDigitalLibrary.org/conference-proceedings-of-spie](https://spiedigitallibrary.org/conference-proceedings-of-spie)

## Scintillation reduction in multi-Gaussian Schell-model beams propagating in atmospheric turbulence

O. Korotkova, S. Avramov-Zamurovic, C. Nelson, R. Malek-Madani, Y. Gu, et al.

O. Korotkova, S. Avramov-Zamurovic, C. Nelson, R. Malek-Madani, Y. Gu, G. Gbur, "Scintillation reduction in multi-Gaussian Schell-model beams propagating in atmospheric turbulence," Proc. SPIE 9224, Laser Communication and Propagation through the Atmosphere and Oceans III, 92240M (3 October 2014); doi: 10.1117/12.2062601

**SPIE.**

Event: SPIE Optical Engineering + Applications, 2014, San Diego, California, United States

# Scintillation reduction in multi-Gaussian Schell-model beams propagating in atmospheric turbulence

O. Korotkova<sup>\*a</sup>, S. Avramov-Zamurovic<sup>b</sup>, C. Nelson<sup>c</sup>, R. Malek-Madani<sup>d</sup>, Y. Gu<sup>e</sup>, G. Gbur<sup>f</sup>

<sup>a</sup>Department of Physics, University of Miami, Coral Gables FL, USA 33146; <sup>b</sup>Weapons and Systems Engineering Department of Mathematics, US Naval Academy, 105 Maryland Avenue, Annapolis, MD 21402, USA <sup>c</sup>Electrical and Computer Engineering Department, US Naval Academy, 105 Maryland Avenue, Annapolis, MD 21402, USA, <sup>d</sup>US Naval Academy, 589 McNair Rd, Stop 10M, Annapolis MD 21402, USA, <sup>e</sup>Optical Remote Sensing Laboratory, Department of Electrical Engineering, The City College of the City University of New York, New York, New York 10031, USA; <sup>f</sup>Department of Physics and Optical Science, University of North Carolina in Charlotte, Charlotte, North Carolina 28223, USA.

## ABSTRACT

The recently introduced class of Multi-Gaussian Schell-model [MGSM] beams is investigated via simulations and experiments with regards to its intensity fluctuations on propagation in atmospheric turbulence. The results indicate that the scintillation index of the MGSM beam is reduced for high values of the summation index, in agreement with previous theoretical results.

**Keywords:** Atmospheric turbulence; random beams; scintillation.

## 1. INTRODUCTION

The ability of beams radiated by random sources to mitigate scintillations (intensity fluctuations) on their propagation in turbulent atmosphere has been known for several decades via theoretical results and computer simulations<sup>1-5</sup> and measurements<sup>6-7</sup>. In all the aforementioned references the tested beams are supposed to be produced by Gaussian Schell-model sources, i.e. assume Gaussian profiles for intensity and degree of coherence, the latter being dependent only on distance between the spatial positions in the source beam cross-section<sup>8</sup>. Recently a variety of random beams with non-Gaussian correlations have been introduced<sup>9</sup> for which the statistical properties vary on propagation in free space and random media not only as a function of typical correlation size but also that of the correlation shape. In particular, it has been shown that pseudo-Bessel-correlated beams and non-uniformly correlated beams can lead to scintillation reduction on propagation in atmospheric turbulence due to their specific correlation shape<sup>10,11</sup>.

Recently a class of random beams with Multi-Gaussian Schell-model [MGSM] correlations that are capable of forming flat intensity profiles in the far-zone of the source in free space and for certain intermediate distances in the atmosphere has been introduced<sup>12-13</sup>. The flatness of the profile is controlled by a number of terms in the sum of Gaussian correlations with different but well-suited weights: the larger this number the flatter the central part of the profile is. The scintillation of the MGSM beams was studied analytically on their propagation in the atmospheric turbulence and it was demonstrated that the scintillation index decreases with increasing number of terms in the Multi-Gaussian function<sup>14</sup>. The purpose of this paper is to confirm, via computer simulations and experimental data collection, the reduction in scintillation in MGSM beams for increasing values of summation parameter.

## 2. THEORETICAL FORMULATION

### 2.1 Multi-Gaussian Schell-model beams

Random sources and beams can be characterized by their cross-spectral density function which depends on two spatial positions and frequency<sup>8</sup>.

\*korotkova@physics.miami.edu; phone 1 (305) 284-6866.

The cross-spectral density function of a scalar Multi-Gaussian source at positions  $\boldsymbol{\rho}_1$  and  $\boldsymbol{\rho}_2$ , at angular frequency  $\omega$  can be characterized by the cross-spectral density function of the form<sup>12, 13</sup>

$$W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \omega) = \exp\left[-\frac{\rho_1^2 + \rho_2^2}{4\sigma^2}\right] \frac{1}{C_0} \sum_{m=1}^M \binom{M}{m} \frac{(-1)^{m-1}}{m} \exp\left[-\frac{(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2}{2m\delta^2}\right], \quad (1)$$

where  $\rho_1 = |\boldsymbol{\rho}_1|$ ,  $\rho_2 = |\boldsymbol{\rho}_2|$ , subscript (0) refers to the source plane, and

$$C_0 = \sum_{m=1}^M \binom{M}{m} \frac{(-1)^{m-1}}{m} \quad (2)$$

is the normalization factor used for obtaining the same maximum intensity level for any number of terms  $M$  in the summation, and  $\binom{M}{m}$  is the binomial coefficient. In Eq. (1)  $\sigma$  is the r.m.s. width of the intensity distribution,  $\delta$  is the

r.m.s. width of the degree of coherence, and the upper index  $M$  relates to the flatness of the intensity profile formed in the far field:  $M = 1$  corresponds to the classical Gaussian Schell-Model source and  $M \rightarrow \infty$  correspond to sources producing far fields with flat centers and abrupt decays at the edges.

## 2.2. Scintillation index

The scintillation index of the random beam propagating to a point  $(\mathbf{r}, z)$  is conventionally defined by the formula<sup>15</sup>:

$$c^2(\mathbf{r}, z) = \langle I^2(\mathbf{r}, z) \rangle / \langle I(\mathbf{r}, z) \rangle^2 - 1, \quad (3)$$

where  $I$  stands for the fluctuating intensity.

## 3. THEORETICAL CALCULATION AND COMPUTER SIMULATIONS

By the mode representation method for Gaussian-Schell model (GSM) beam used in Ref. 16, the cross-spectral density function of the MGSM beam represented by Eq. (1) can be expressed in a different mode representation,

$$W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \omega) = \iint_{-\infty}^{\infty} \psi(\boldsymbol{\rho}_1, \mathbf{K}) \psi^*(\boldsymbol{\rho}_2, \mathbf{K}) d^2K \quad (4)$$

where the modes are given by expressions

$$\psi(\boldsymbol{\rho}, \mathbf{K}) = \frac{1}{c_0} \left\{ \sum_{m=1}^M \binom{M}{m} \frac{(-1)^{m-1}}{m} \sqrt{2m\pi\delta^2} \exp\left(-\frac{mk^2\delta^2 u_1^2}{4}\right) \right\} \exp\left(-\frac{\rho^2}{4\sigma^2}\right) \exp(ik\mathbf{K} \cdot \boldsymbol{\rho}). \quad (5)$$

Eqs. (4) and (5) show that a MGSM beam is an incoherent superposition of collimated Gaussian beams whose transverse wave vectors  $\mathbf{K}$  are random and the associated amplitudes form a flat-top function in the  $\mathbf{K}$  plane. However, the evanescent modes included in Eq. (4) decay fast. Only the propagating modes contribute to the light on the receiver plane. Eq. (5) is rewritten as

$$\psi(\boldsymbol{\rho}, \mathbf{u}_{\perp}) = \frac{1}{c_0} \left\{ \sum_{m=1}^M \binom{M}{m} \frac{(-1)^{m-1}}{m} \sqrt{2m\pi\delta^2} \exp\left(-\frac{mk^2\delta^2 u_1^2}{4}\right) \right\} \exp\left(-\frac{\rho^2}{4\sigma^2}\right) \exp(ik\mathbf{u}_{\perp} \cdot \boldsymbol{\rho}), \quad (6)$$

where  $k$  is wave number and  $\mathbf{u}_{\perp} = (u_{\perp x}, u_{\perp y})$  is the projection of the unit direction vector in the source plane. Since the scintillation properties of a partially coherent beam can be well approximated by a finite number of its constituent modes<sup>10</sup>, we can study the scintillation properties of a MGSM beam through the discretized form of Eq. (4):

$$W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \omega) = \sum_{n=1}^N \psi_n(\boldsymbol{\rho}_1, \mathbf{u}_{\perp n}) \psi_n^*(\boldsymbol{\rho}_2, \mathbf{u}_{\perp n}). \quad (7)$$

The scintillation index of such a pseudo-MGSM beam is written as

$$c^2(\boldsymbol{\rho}, z) = \frac{\sum_p^N \sum_q^N \langle I_p(\boldsymbol{\rho}, z) I_q(\boldsymbol{\rho}, z) \rangle}{\left[ \sum_{p=1}^N \langle I_p(\boldsymbol{\rho}, z) \rangle \right]^2} - 1, \quad (8)$$

where  $I_p(\boldsymbol{\rho}, z)$  is the instantaneous intensity of the  $p$ th mode on the receiver plane. Note that  $N$  and  $M$  are unrelated:  $M$  is a parameter that characterizes the flatness of the MGSM's coherence function, while  $N$  is the total number of propagating modes used in Eq. (7). From Eq. (6), it can be seen that  $M$  is used in each propagating mode. Using the expressions of  $\langle I_p(\boldsymbol{\rho}, z) \rangle$  and  $\langle I_p(\boldsymbol{\rho}, z) I_q(\boldsymbol{\rho}, z) \rangle$  which are available in Ref. 10, we can analyze the scintillation properties of MGSM beams in weak turbulence by numerical evaluation of  $\langle I_p(\boldsymbol{\rho}, z) \rangle$  and  $\langle I_p(\boldsymbol{\rho}, z) I_q(\boldsymbol{\rho}, z) \rangle$ .

In general, the projection of the unit direction vector  $\mathbf{u}_\perp$  is confined in a circular area  $|\mathbf{u}_\perp| \leq 1$ . However, for given turbulence and propagation parameters, the scintillation index of a MGSM beam can be represented by a pseudo one whose  $\mathbf{u}_{\perp n}$  is sampled in a small circular area with a finite separation  $\Delta u_\perp$ . Because a pseudo MGSM beam requires sampling of  $\mathbf{u}_{\perp n}$  in a two dimensional plane, it yields a large amount of constituent modes followed by an intensive computation of the scintillation index through Eq. (7). We start with a one dimensional MGSM beam, namely  $u_{\perp y} = 0$ .

Figure 1 presents the on-axis scintillation index of a one dimensional MGSM beam propagating through a certain atmospheric channel as a function of the upper index  $M$ . For the theoretical curve, calculated from Eq. (6),  $|u_{\perp x}| \leq 5 \times 10^{-5}$  and  $\Delta u_{\perp x} = 2.5 \times 10^{-7}$ , and hence 401 modes are used (indeed,  $(2 \cdot 5 \cdot 10^{-5}) / (2.5 \cdot 10^{-7}) + 1 = 401$ ). A plot of the scintillation index by the independent phase screen simulation method<sup>17</sup> is also shown for comparison. For the simulation plot  $|u_{\perp x}| \leq 3 \times 10^{-5}$  and  $\Delta u_{\perp x} = 10^{-6}$ , - 61 modes are used ( $(2 \cdot 3 \cdot 10^{-5}) / (10^{-6}) + 1 = 61$ ). The relative difference between these two plots is between 1% and 2%, where the difference is defined as  $(c_{\text{sim}}^2 - c_{\text{theory}}^2) / c_{\text{sim}}^2$ . Thus the results of simulation and of the theory are in good agreement. Figure 1 also shows that it is sufficient to use a small set of modes to study the scintillation properties of the MGSM beams.

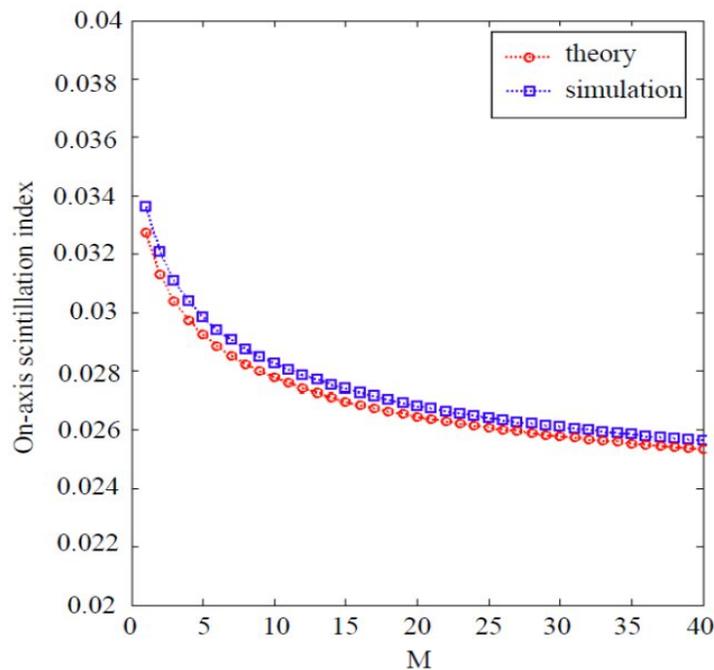


Figure 1. On-axis scintillation index vs.  $M$ . Parameters  $\lambda = 1.55 \mu\text{m}$ ,  $\delta = 3.54 \text{ cm}$ ,  $\sigma = 2.5 \text{ cm}$ ,  $L = 2 \text{ km}$ ,  $C_n^2 = 10^{-15} \text{ m}^{-2/3}$ , von Karman spectrum with  $L_0 = 10 \text{ m}$  and  $l_0 = 1 \text{ mm}$ .

Figure 2 presents the dependence of the on-axis scintillation index of the propagating beam on the coherence width normalized by the r.m.s. beam width for several values of  $M$ . From the analysis of one dimensional MGSM beam shown in Figures 1 and 2, we can use sampling range  $\sqrt{u_{\perp x}^2 + u_{\perp y}^2} \leq 2.4 \times 10^{-5}$  and sampling separation  $\Delta u_{\perp x} = \Delta u_{\perp y} = 0.6 \times 10^{-6}$  to generate a two dimensional pseudo MGSM beam and study its scintillation properties for the given turbulence propagation parameters. In Fig. 3, the scintillation indices of the GSM ( $M=1$ ) and MGSM ( $M=20$ ) beams are two-dimensional, discretized. 49 modes are used to generate such MGSM and GSM ( $M=1$ ) beams. . They are

obtained by the phase screen simulation which is applicable when turbulence is moderate to strong. It can be seen that MGSM beams outperform both fully coherent Gaussian beams and the GSM beams with comparable coherence length.

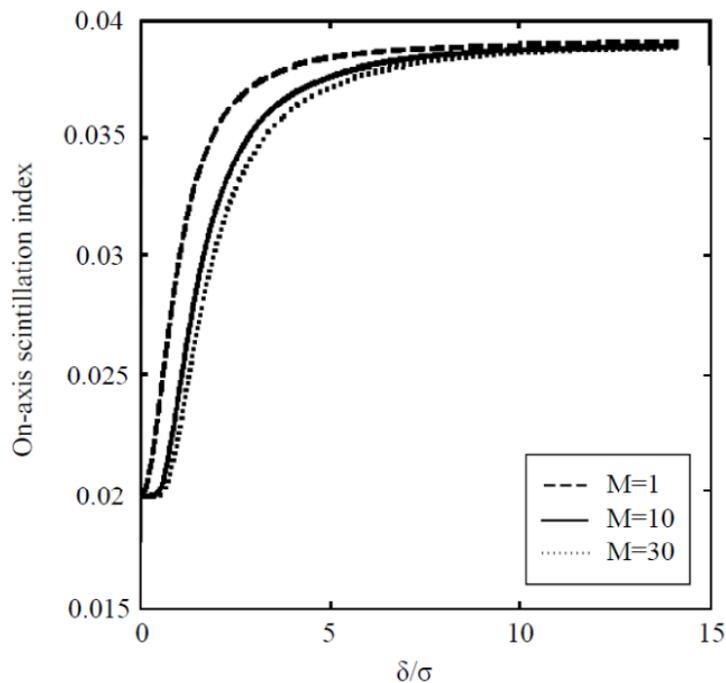


Figure 2. On-axis scintillation index vs. relative coherence length  $\delta/\sigma$ . The other parameters are the same as in Fig. 1.

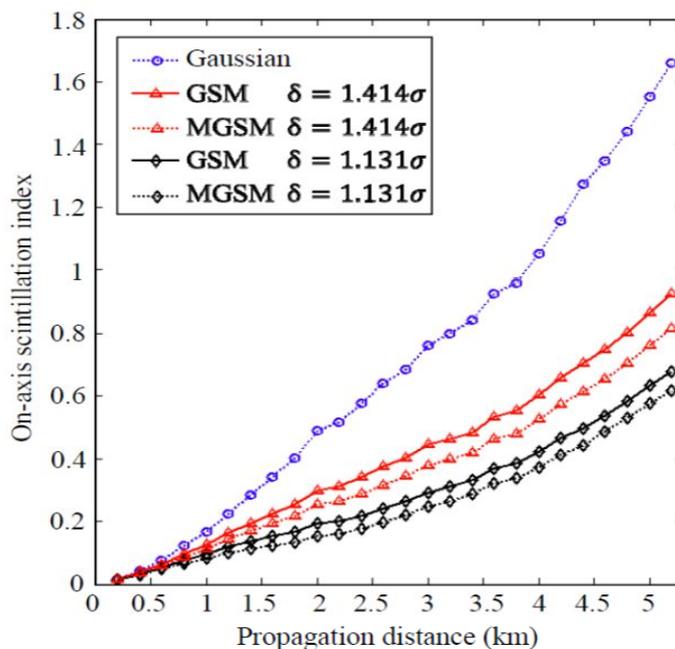


Figure 3. On-axis scintillation index vs. propagation distance. The other parameters are the same as in Fig. 1 except  $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ .

## 4. EXPERIMENTAL RESULTS

### 4.1 Generation procedure of Multi-Gaussian Schell-model beams

A reflective nematic Spatial Light Modulator (SLM) with resolution 256x256 pixels was used to create phase screen realizations compatible with MGSM phase correlation. A similar MATLAB program that was used by the group for generation of the Gaussian Schell-model beams with variable coherence state<sup>7</sup> has been adopted for the Multi-Gaussian Schell-model beams by modifying the window function. To minimize the influence of light diffraction from the edges of SLM, the reflected beam has been spatially limited by using a mechanical iris with a 1 cm initial propagation diameter.

For the phase screen generation, an alternative measure of the r.m.s. phase correlation is used directly providing the value in terms of the number of pixels rather than in millimeters. In order to relate the two parameters the following scaling law is used

$$\delta = \frac{L}{N} \times \gamma_{\phi}, \quad (9)$$

where  $L$  stands for the linear dimension of the (square-shaped) SLM and  $N$  stands for the number of pixels in one row. For example, for the case of  $\gamma_{\phi}^2 = 128$  that was used in the experiment the approximate r.m.s. phase correlation size,  $\delta$ , is computed as follows: the SLM array has 256x256 pixels over the area of 6.14 mm by 6.14 mm and hence  $\delta = \frac{6.14\text{mm}}{256} \times \sqrt{128} = 0.27\text{mm}$ .

In order to eliminate the effect of the zero-order bright spot in the center of the random beam reflected from the SLM we have multiplied the window function by a cosine function with a suitable argument. Such a procedure creates two bright-spot-free replicas of the beam at locations off the optical axis one of which is used for propagation.

### 4.2. Experiment

The field test campaign took place in July, 2014 at the US Naval Academy. A link of 230 m was set up over the water (see Figure 1). The tests were performed during night in calm weather conditions. A low power, 2 mW, red ( $\lambda=630$  nm) He-Ne laser beam was used at the source. A beam expander was used to adjust the beam size in order to use the whole area of the SLM. At the receiver system the laser light was observed using a camera with the recording rate of 10 frames per second and sensitivity of possible 256 values of intensity. Since the MGSM beams produce flat top profile the camera was pointed into the light direction in the area of the flat top part of the beam. The beam was collected directly on the surface of the camera sensor by using the neutral density and red notch filters to have the optimal range of recorded intensity values. Light sensor was 7.62 x 7.62 mm in size, with a resolution of 1024x1280 pixels, with an area of 4.65 $\mu\text{m}^2$  per pixel. The camera resolution provided 1.3 million readings that were used to find the average scintillation index.



Figure 4 Experimental setup for generation and propagation of MGSM beams in atmospheric turbulence. Left: transmitter side; Right: receiver side.

### 4.3 Scintillation index analysis methodology

The objective of the campaign was to determine the scintillation index dependence on the index  $M$ . The tests were carried out for  $M$  values of 1, 4, 10, 15, 20, 25, 30, 35 and 40. To generate MGSM beams screens were created for SLM with the window function given by Eq. (1). Spatial coherence level was determined by the correlation width set to 128. The screens in the SLM were cycled at the rate of 333 Hz. The camera exposure time was approximately 50 ms, effectively averaging over 16 realizations of the source.

To observe the influence of the atmosphere on beam propagation each observation test lasted two minutes creating several thousand image realizations. Scintillation index was calculated for each valid pixel for a time series reflecting the duration of the test: 1.3 million scintillation index values were used for each data point. This approach provided insight into precision and reliability of the measurements.

### 4.4 Data analysis

Three consecutive sets of data were taken and referred to as Runs 1, 2 and 3 on Fig. 5. The slight difference between the runs, especially for low values of  $M$ , stems from slightly different atmospheric conditions. The data clearly indicates that the same trend as given by analytical calculations and computer simulations is present: as  $M$  grows the scintillation index reduces. Somewhat different values of the scintillation index for low values of  $M$  ( $M = 1$  and  $M = 4$ ) among the three sets occurred because for these measurements the camera gain was adjusted in order to eliminate data saturation.

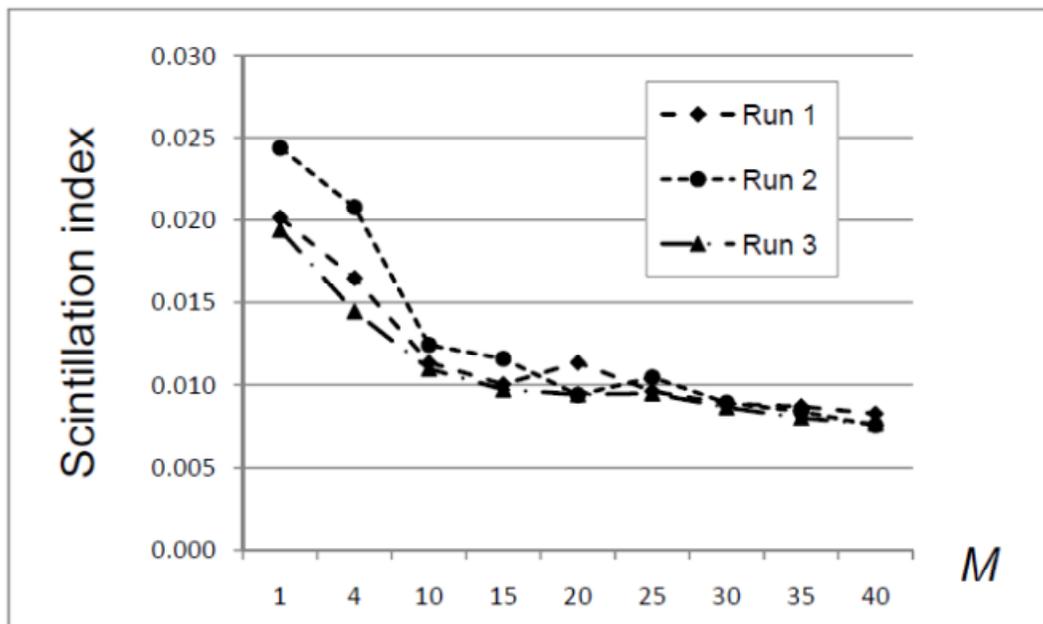


Figure 5 Experimental results on the scintillation index of MGSM beams propagating in the atmospheric turbulence as a function of upper index  $M$ .

## 5. CONCLUSIONS

The recent theoretical result on the reduction of the scintillation index of the MGSM beam on propagation in turbulent atmosphere by increasing values of flatness parameter  $M$  is confirmed by computer simulations and experiments. The MGSM sources were generated via reflection from the nematic SLM cycling at a high rate. Because the intensity profile of the received beam was flat the scintillation index was not influenced by the edge profile and it was calculated by averaging over the whole collection of camera pixels.

## 6. ACKNOWLEDGEMENTS

O. Korotkova's, S. Avramov-Zamurovic, C. Nelson and R. Malek-Madani's work is supported by grant: US ONR (N00189-14-T-0120) O. Korotkova's work is also supported by grant: US AFOSR (FA9550-12-1-0449). G. Gbur's research is sponsored by grant: US AFOSR (FA9550-13-1-0009). The authors are obliged to Stephen Guth for helping during the field testing and processing of the data.

## REFERENCES

- [1] Leader, J. C. "Intensity fluctuations resulting from partially coherent light propagating through atmospheric turbulence," *J. Opt. Soc. Am.* 69, 73-83 (1979).
- [2] Banakh, V., Buldakov V. M., and Mironov, V. L. "Intensity fluctuations of a partially coherent light beam in a turbulent atmosphere," *Opt. Spectrosc.* 54, 1054-1059 (1983).
- [3] Ricklin, J. C. and Davidson, F. M., "Atmospheric turbulence effects on a partially coherent Gaussian Beam: implications for free-space laser communication," *J. Opt. Soc. Am A* 19, 1794-1802 (2002).
- [4] Korotkova, O., Andrews, L. C. and Phillips, R. L., "A model for partially coherent Gaussian beam in atmospheric turbulence with applications in lasercom," *Opt. Eng.* 43, 330-341 (2004).
- [5] Schulz, T. J., "Optimal beams for propagation through random media," *Opt. Lett.* 30, 1093-1095 (2005).
- [6] Efimov, A., Velizhanin, K., Gelikonov, G. "Simultaneous scintillation measurements of coherent and partially coherent beams in an open atmosphere experiment," *Proc. SPIE* 8971, 897105 (2014).
- [7] Avramov-Zamurovic, S., Korotkova, O., Nelson C., and Malek-Madani, R., "The dependence of the intensity PDF of a random beam propagating in the maritime atmosphere on source coherence," *Waves in Complex and Random media* 24, 69-82 (2014).
- [8] Mandel, L. and Wolf. E., [Optical Coherence and Quantum Optics], Cambridge University Press, Cambridge (1995).
- [9] Korotkova O., [Random beams: theory and applications], CRC Press, Boca Raton, FL, (2013).
- [10] Gu, Y., and Gbur, G., "Scintillation of pseudo-Bessel correlated beams in atmospheric turbulence," *J. Opt. Soc. Am. A* 27, 2621-2629 (2010).
- [11] Gu, Y. and Gbur, G., "Scintillation of non-uniformly correlated beams in atmospheric turbulence", *Opt. Lett.* 38, 1395-1397 (2013).
- [12] Sahin, S. and Korotkova, O., "Light sources generating far fields with tunable flat profiles," *Opt. Lett.* 37, 2970-2972 (2012).
- [13] Korotkova, O., Sahin, S. and Shchepakina, E., "Multi-Gaussian Schell-model beams," *J. Opt. Soc. Am. A* 29, 2159- 2164 (2012).
- [14] Yuan, Y., Liu, X., Wang, F., Chen, Y., Cai, Y., Qu, J., Eyyuboglu, H.T., "Scintillation index of a multi-Gaussian Schell-model beam in turbulent atmosphere," *Opt. Commun.*, 305, 57-65 (2013).
- [15] Andrews, L. C. and Phillips, R. L., [Laser beam propagation in random media], SPIE press, Bellington, Washington (2005).
- [16] Gan, C. H., Gu, Y., Gbur, G. and Visser, T. D. "Coherence converting plasmonic hole arrays," *Plasmonics* 7, 313-322 (2012).
- [17] Schmidt, J. D., [Numerical simulation of optical beam propagation with examples in MATLAB], SPIE Press, Bellington, Washington (2010).