Measurements and comparison of the probability density and covariance functions of laser beam intensity fluctuations in a hot-air turbulence emulator with the maritime atmospheric environment

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Office of Naval Research, Counter Directed Energy Weapons Program
The Johns Hopkins Applied Physics Laboratory and
The Johns Hopkins University
**FIELD TESTS – OVERVIEW**

- Field Tests off of Atlantic Coast and at USNA
- 5 – 18 km optical horizon off of Atlantic Coast and 650 m at USNA
Turbulence Generator
Mk I, Mod 0

Photo looking down through turbulence generator, daughter Aurora pictured
**Initial Turbulence Emulator**

- Hot and cool air meet and mix
- Red Oak, aluminum flashing, modular

**Theoretical Results**

- High delta $T \rightarrow$ High $C_n^2$

\[
C_n^2 = (77.8 \times 10^{-6} \frac{P}{T^2})^2 C_T^2
\]

\[
C_T^2 = \frac{\langle (T_1 - T_2)^2 \rangle}{R^{2/3}}
\]

\[
\sigma_R^2 = 1.23 C_n^2 k^6 L^{11/6}, \text{Rytov variance}
\]

\[
\sigma_I^2 = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2}, \text{scintillation index}
\]

**Results**

- $C_n^2 \sim 10^{-9} \ m^{-2/3}$, $\sigma_I^2 \sim 0.02 \ - \ 0.05$, $\sigma_R^2 \sim 0.1 \ - \ 0.7$

Initial design showed some positive results/proof of concept
2\textsuperscript{ND} TURBULENCE EMULATOR

MOD 0 TO THE MOD 1
TE COMPARISON WITH FIELD TESTS

1.6 mm dia. 1550 nm fiber coll., no AO, \( N_f = 0.2 \)

1550 nm, 10 cm Tx, closed loop, AO, 10.7 km, \( N_f = 0.2 \)

CMOS, HeNe, Stat. SLM, HeNe, 650 m, Expander, \( N_f = 0.04 \)

Fiber coll., 1550 nm pol., exp., SLM, Mech. IRIS, \( N_f = 0.04 \)

Modified set-ups
**THEORY**

- **Gamma-Laguerre (GL)**
  - Medium and source independent
  - Uses first n moments
- **Gamma-Gamma (GG)**
  - Source, medium dependent
  - Uses first two moments
- **Lognormal (LN)**
  - Classical weak turbulence model
- **Temporal Autocovariance**

### \[ W_{GL}(I) = W_g(I) \sum_{n=0}^{\infty} W_n L_n^{(\beta-1)} \left( \frac{\beta I}{\mu} \right), I \geq 0 \]

- \[ W_g(I) = \frac{1}{I^\beta (\beta I/\mu)^\beta} \exp \left[ - \frac{\beta I}{\mu} \right] \]
- \[ \mu = \langle I \rangle, \quad \beta = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I^2 \rangle} \]
- \[ W_n = n! \Gamma(\beta) \sum_{k=0}^{n} \frac{(-\beta/\mu)^k \langle I^k \rangle}{k!(n-k)! \Gamma(\beta+k)} \]
- \[ W_0 = 1, W_1 = W_2 = 0 \]
- \[ I_n^{(\beta-1)}(x) = \sum_{k=0}^{n} \left( \frac{n + \beta - 1}{n - 1} \right)^{\frac{1}{k!}} \]

### \[ W_{GG}(I) = \frac{2(\alpha \beta)^{\frac{a+\beta}{2}}}{\Gamma(\alpha) \Gamma(\beta)} I^{-\frac{a+\beta}{2}} \left( 2\sqrt{\alpha \beta I} \right)^{a-\beta}, I > 0 \]

- \[ \alpha = \frac{1}{\exp(\sigma_{inx}^2)-1}, \quad \beta = \frac{1}{\exp(\sigma_{lny}^2)-1} \]
- \[ \sigma_{inx}^2 = \frac{0.49 \sigma_B^{12}}{[1+0.56(1+\theta)\sigma_B^{12}]^{7/6}} \quad \text{and} \quad \sigma_{lny}^2 = \frac{0.51 \sigma_B^{12}}{[1+0.69(1+\theta)\sigma_B^{12}]^{5/6}} \]
- \[ \sigma_B^2 = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2} \]
- \[ \theta = [1 + \left( \frac{2L}{k W_0^2} \right)^2]^{-1} \]

### \[ W_{LN}(I) = \frac{1}{I \sigma \sqrt{2\pi}} \exp \left[ - \frac{[\ln(I) - \mu]^2}{2\sigma^2} \right], I > 0 \]

- \[ \mu = \langle \ln(I) \rangle \]
- \[ \sigma^2 = \text{var}(\ln(I)) \]
TE COMPARISON WITH FIELD TEST

Similar PDFs, different Autocovariances
- Autocovariance could relate to overall length of fades
- $D_{Det}/\rho_0$ ratio might give information about overall # fades

<table>
<thead>
<tr>
<th>Case</th>
<th>$N_f$</th>
<th>$D_g/\rho_0$</th>
<th>$\sigma_I^2$</th>
<th>$T_I$</th>
<th># Fades</th>
<th>80/100 ms</th>
<th>Ch. Avail.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.2</td>
<td>2</td>
<td>0.12</td>
<td>9.5</td>
<td>552</td>
<td>7/34</td>
<td>96.7%</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>2</td>
<td>0.13</td>
<td>8.7</td>
<td>1717</td>
<td>1/31</td>
<td>96.4%</td>
</tr>
<tr>
<td>C</td>
<td>0.2</td>
<td>1</td>
<td>0.13</td>
<td>1.2</td>
<td>1820</td>
<td>1/13</td>
<td>97.1%</td>
</tr>
</tbody>
</table>
**TE COMPARISON WITH FIELD TEST**

*USNA CMOS*

Hist (Red Dots •)  
GL – Black line  
LN (Green – – –)

Hist (Red Dots •)  
GL – Black line  
LN (Green – – –)

<table>
<thead>
<tr>
<th>Case</th>
<th>$N_f$</th>
<th>$D_S/\rho_0$</th>
<th>$\sigma_1^2$</th>
<th>$T_1$</th>
<th># Fades</th>
<th>80/100 ms</th>
<th>Ch. Avail.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.04</td>
<td>0.1</td>
<td>0.014</td>
<td>0.3</td>
<td>3384</td>
<td>1/1</td>
<td>96.5%</td>
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<tr>
<td>B</td>
<td>0.04</td>
<td>0.1</td>
<td>0.014</td>
<td>1.5</td>
<td>2092</td>
<td>1/6</td>
<td>96.7%</td>
</tr>
</tbody>
</table>

- Similar PDFs, different Autocovariances
  - Autocovariance could relate to overall length of fades
  - $D_{Det}/\rho_0$ ratio might give information about overall # fades
CONCLUSIONS/DISCUSSION

- Hot-air turbulence emulator showed a close comparison of $\sigma_I^2$, $N_f$, and $D_S/\rho_0$ with a corresponding closeness in PDFs.
- The 2nd order statistics through the temporal autocovariance function and $T_1$ showed significant differences.
  - Having a smaller $T_1$ could indicate shorter fade lengths.
- A small $D_{det}/\rho_0$ ratio could indicate a higher number of fades.


