Measurements and comparison of the probability density and covariance functions of laser beam intensity fluctuations in a hot-air turbulence emulator with the maritime atmospheric environment

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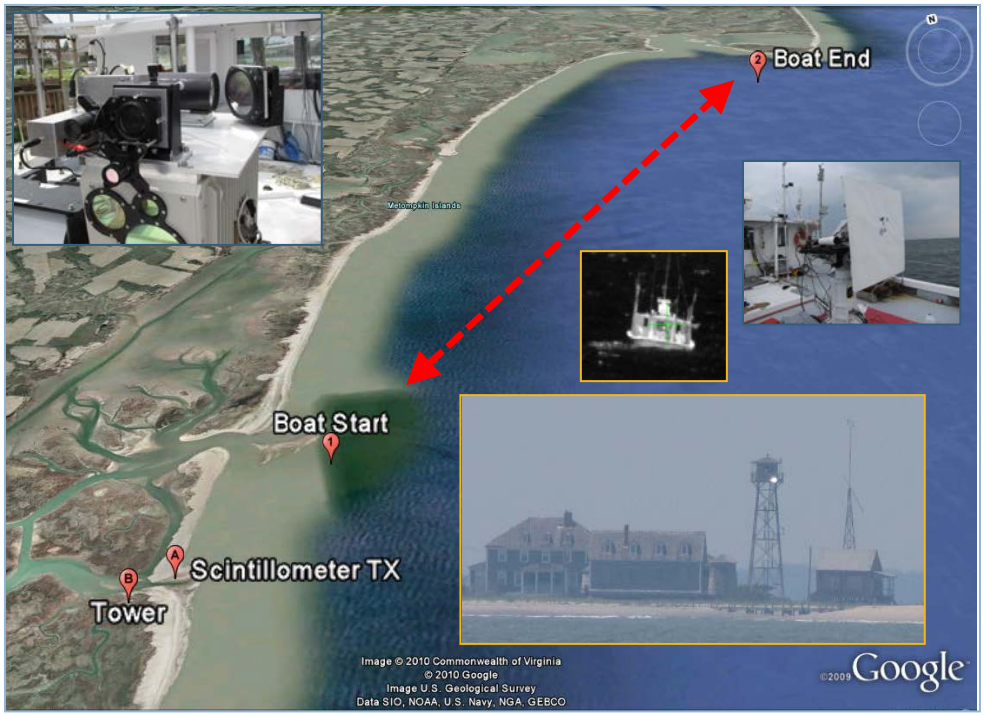
The Johns Hopkins University

Field Tests – Overview

- Field Tests off of Atlantic Coast and at USNA
- 5 – 18 km optical horizon off of Atlantic Coast and 650 m at USNA

Movie Clip



Turbulence Generator Mk I, Mod 0

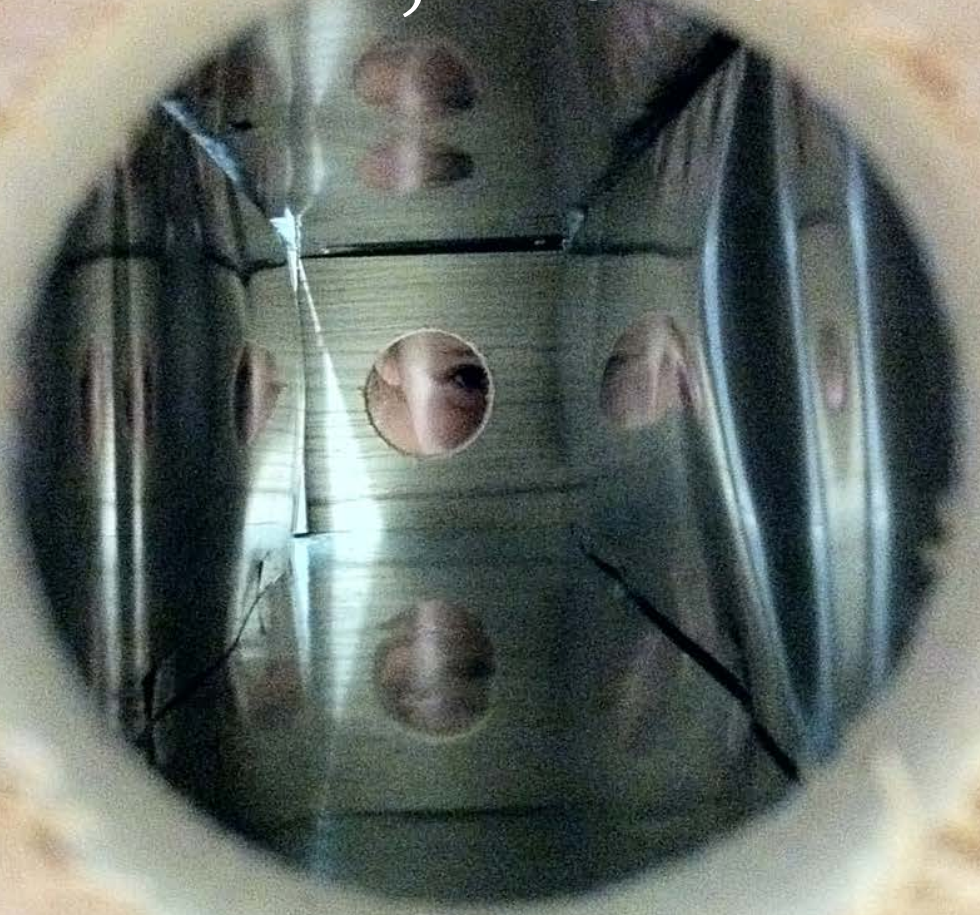
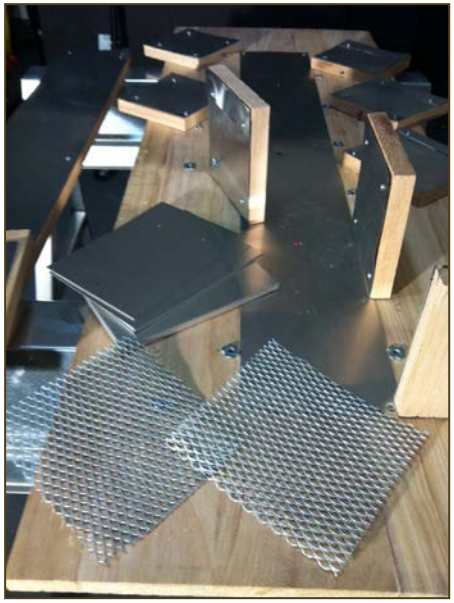
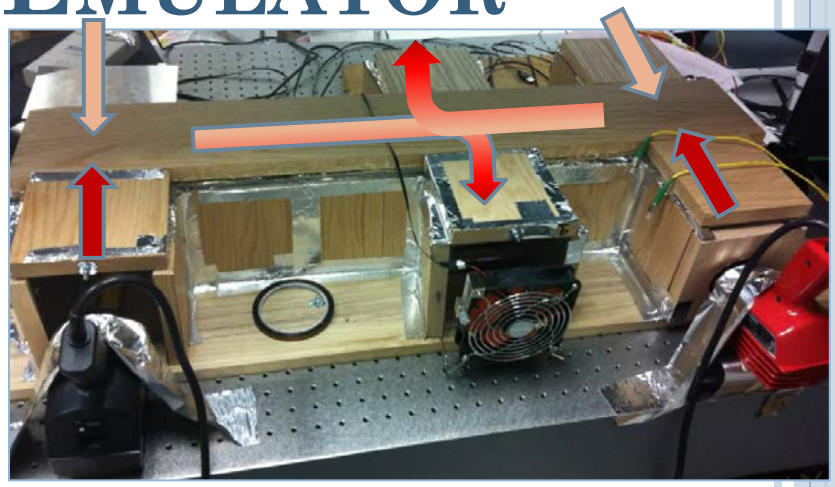


Photo looking down through turbulence generator, daughter Aurora pictured

Initial Turbulence Emulator





- Hot and cool air meet and mix
- Red Oak, aluminum flashing, modular

Theory

High delta T → High C_n^2

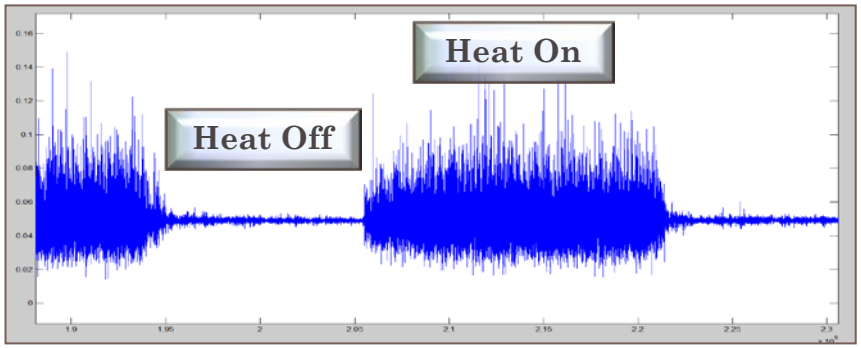
$$C_n^2 = (77.8 * 10^{-6} \frac{P}{T^2})^2 C_T^2$$

$$C_T^2 = \frac{\langle (T_1 - T_2)^2 \rangle}{R^{2/3}}$$

$$\sigma_R^2 = 1.23 C_n^2 k^{\frac{7}{6}} L^{\frac{11}{6}}, Rytov\ variance$$

$$\sigma_I^2 = \frac{(\langle I^2 \rangle - \langle I \rangle^2)}{\langle I \rangle^2}, scintillation\ index$$

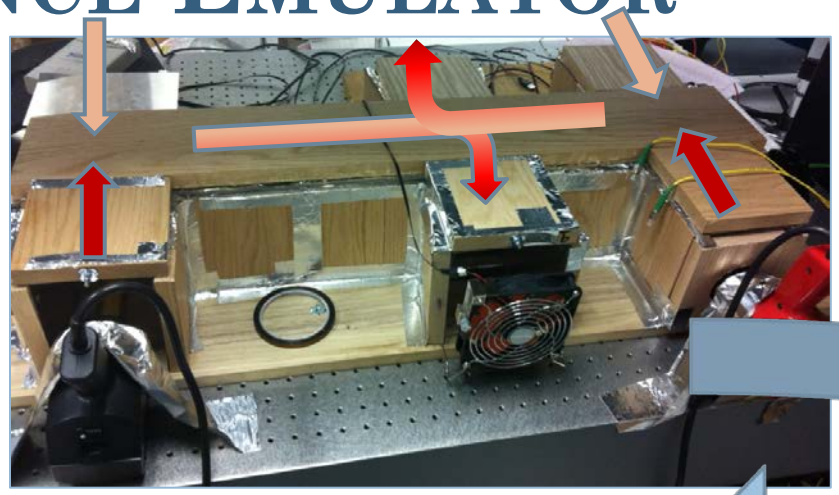
Results



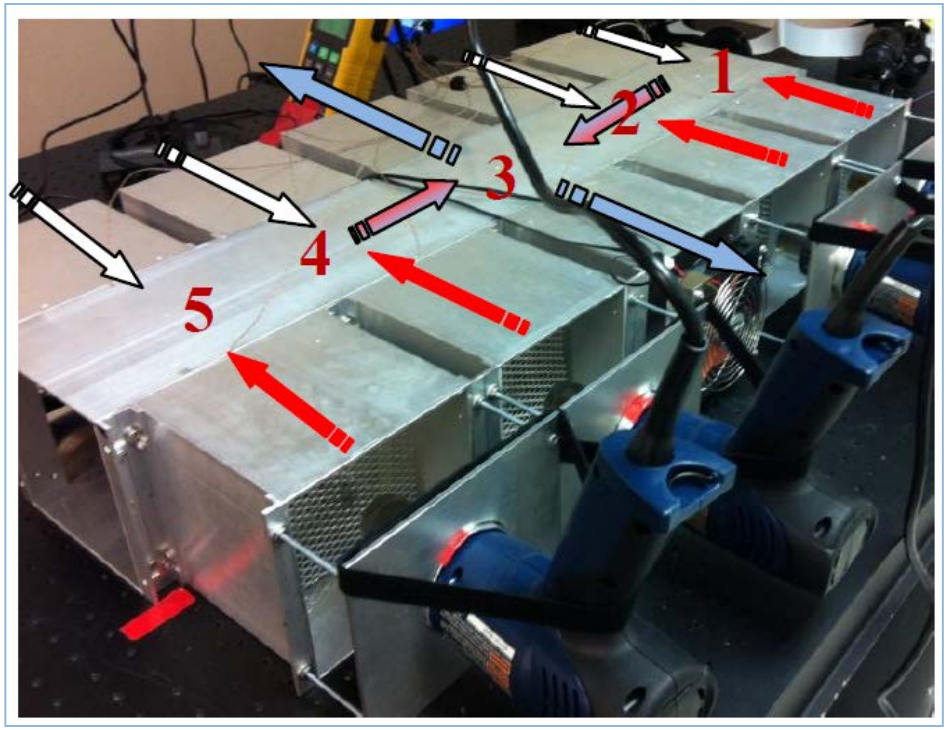
C_n^2 ~10^{-9} $m^{-2/3}$, σ_I^2 ~ 0.02 – 0.05, σ_R^2 ~0.1 - 0.7

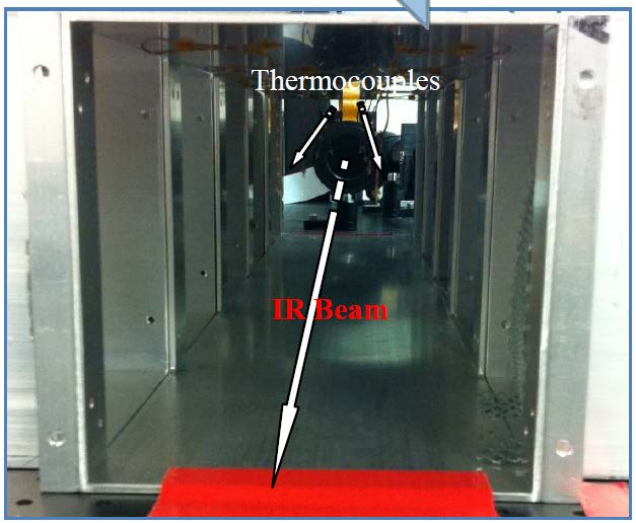
Initial design showed some positive results/proof of concept

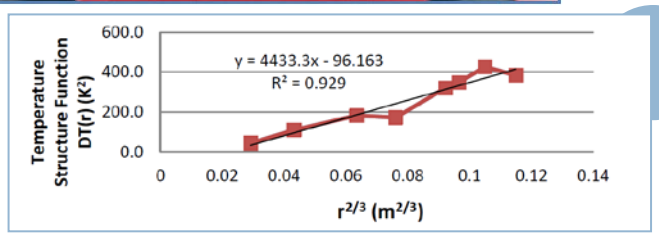
2ND TURBULENCE EMULATOR



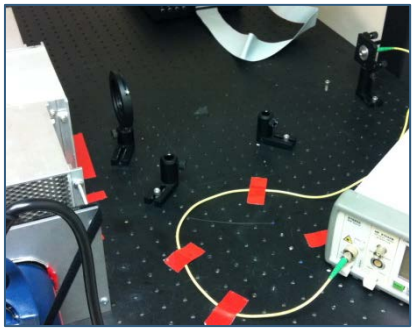
MOD 0 TO THE MOD 1



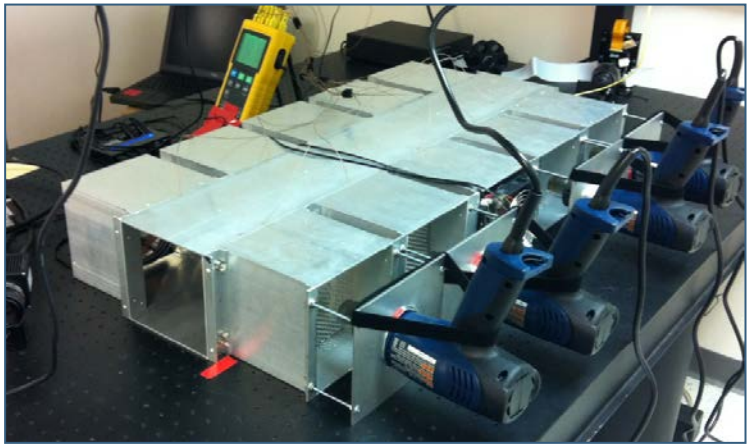


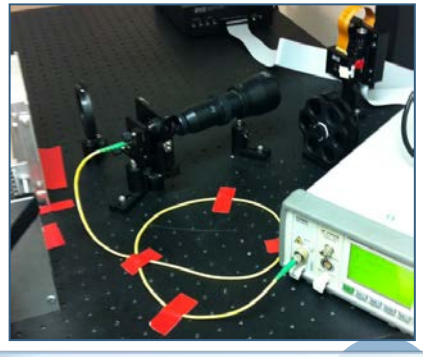


TE comparison with Field Tests



1.6 mm dia. 1550 nm fiber coll., no AO, N_f = 0.2





Fiber coll., 1550 nm pol., exp., SLM, Mech. IRIS, N_f = 0.04

Modified set-ups

Theory

- Gamma-Laguerre (GL)
  - Medium and source independent
  - Uses first n moments
- Gamma-Gamma (GG)
  - Source, medium dependent
  - Uses first two moments
- Lognormal (LN)
  - Classical weak turbulence model
- Temporal Autocovariance

$$B_x(t_1,t_2) = \left\langle \left[x(t_1) - \left\langle x(t_1)\right\rangle\right]\left[x(t_2) - \left\langle x(t_2)\right\rangle\right]\right\rangle$$

- $B_x(\tau) = R_x(\tau) - m^2$ and $R_x(\tau) = \left\langle x(t_1)x(t_2)\right\rangle$
- $B_{(\exp fit)}(t_{data}) = Ae^{(-t_{data}/T_1)}$

$$W_{GL}(I) = W_g(I)\sum_{n=0}^{\infty} W_n L_n^{(\beta-1)}\left(\frac{\beta I}{\mu}\right), I \geq 0$$

- $W_g(I) = \frac{1}{\Gamma(\beta)}\left(\frac{\beta}{\mu}\right)^{\beta} I^{\beta-1}\exp\left(-\frac{\beta I}{\mu}\right)$
- $\mu = \langle I\rangle,\ \beta = \frac{\langle I\rangle^2}{(\langle I^2\rangle - \langle I\rangle^2)}$
- $W_n = n!\,\Gamma(\beta)\sum_{k=0}^{n}\frac{(-\beta/\mu)^k\langle I^k\rangle}{k!(n-k)!\Gamma(\beta+k)},$
- $W_0 = 1, W_1 = W_2 = 0$
- $L_n^{(\beta-1)}(x) = \sum_{k=0}^{n}\binom{n+\beta-1}{n-1}\frac{(-x)^k}{k!}$

$$W_{GG}(I) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} I^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta I}\right), I > 0$$

- $\alpha = \frac{1}{\exp\left(\sigma_{lnx}^2\right)-1},\ \beta = \frac{1}{\exp\left(\sigma_{lny}^2\right)-1}$
- $\sigma_{lnx}^2 = \frac{0.49\sigma_B^2}{[1+0.56(1+\vartheta)\sigma_B^{\frac{12}{5}}]^{7/6}}$ *and* $\sigma_{lny}^2 = \frac{0.51\sigma_B^2}{[1+0.69(1+\vartheta)\sigma_B^{\frac{12}{5}}]^{5/6}}$
- $\sigma_B^2 = (\langle I^2\rangle - \langle I\rangle^2)/\langle I\rangle^2,$
- $\vartheta = [1 + (\frac{2L}{kW_0^2})^2]^{-1}$

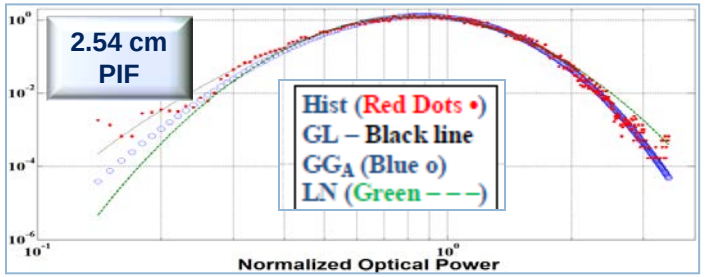
$$W_{LN}(I) = \frac{1}{I\sigma\sqrt{2\pi}} exp\left[-\frac{[\ln(I)-\mu]^2}{2\sigma^2}\right], I > 0$$

- $\mu = \langle \ln(I)\rangle$
- $\sigma^2 = var(\ln(I))$

TE comparison with Field Test

Atlantic Coast

A



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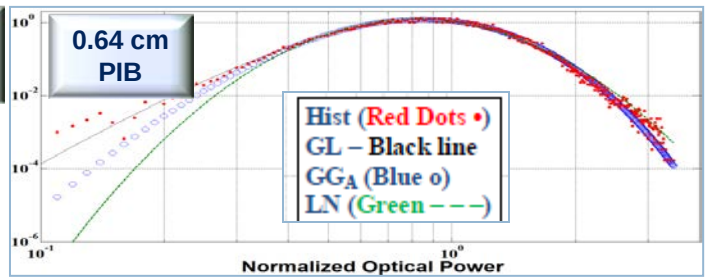

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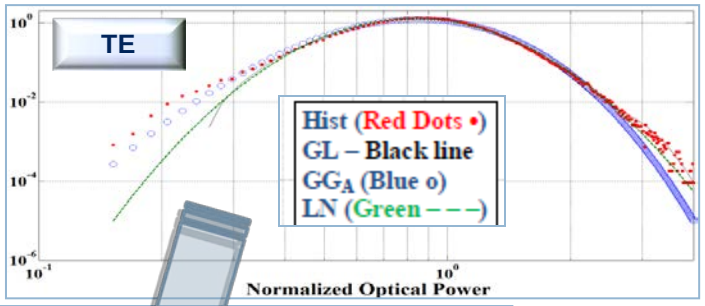
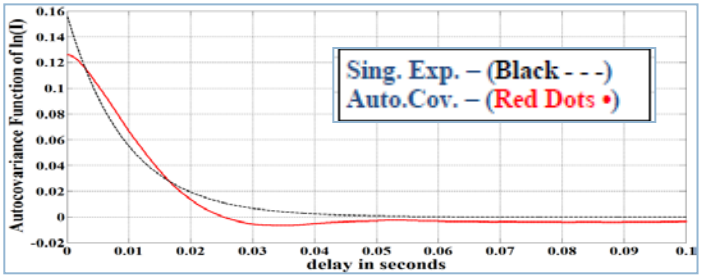
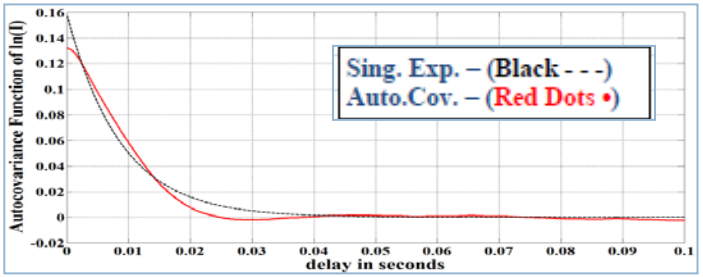
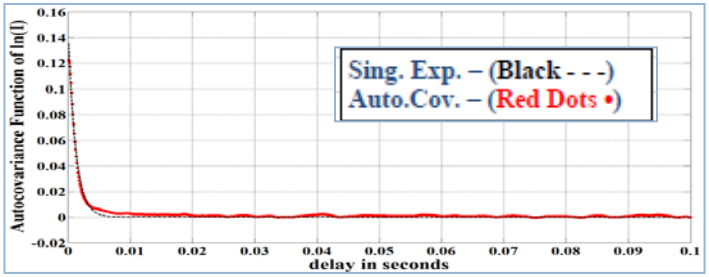
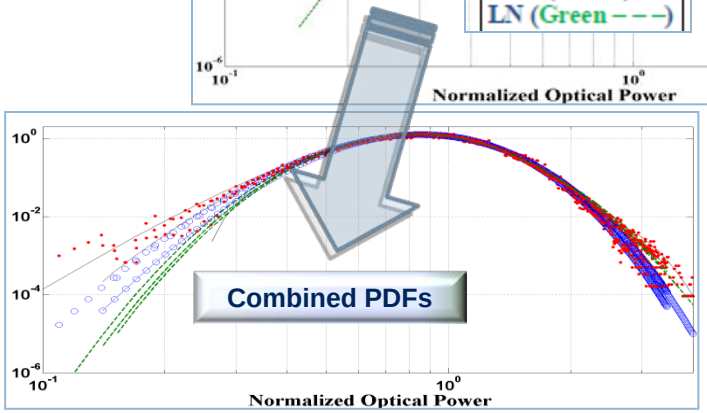








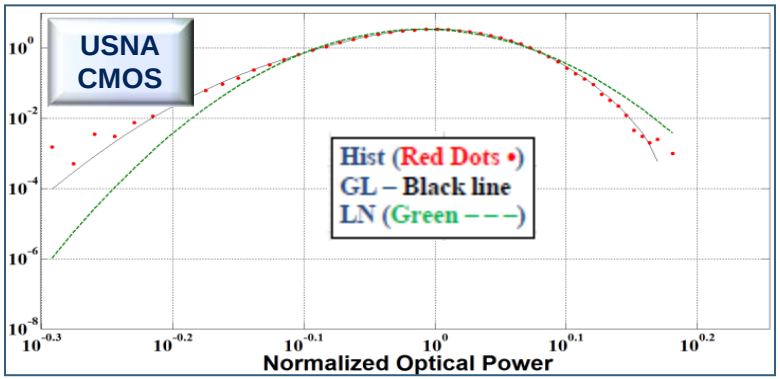


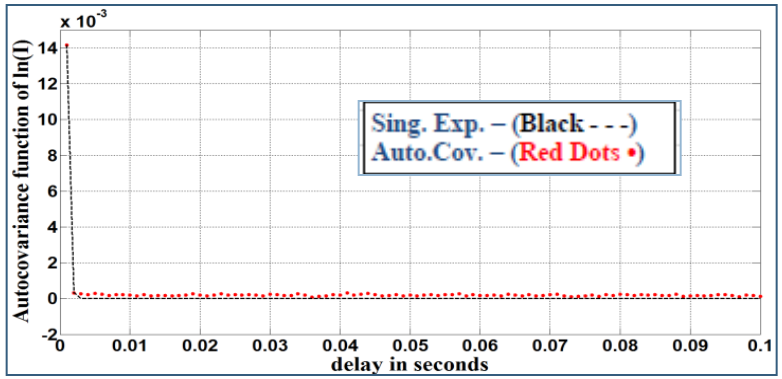
Case	N_f	D_S/ρ_0	σ_I^2	T_1	# Fades	80/100 ms	Ch. Avail.
A	0.2	2	0.12	9.5	552	7/34	96.7%
B	0.2	2	0.13	8.7	1717	1/31	96.4%
C	0.2	1	0.13	1.2	1820	1/13	97.1%

- Similar PDFs, different Autocovariances
  - Autocovariance could relate to overall length of fades
  - D_{Det}/ρ_0 ratio might give information about overall # fades

TE comparison with Field Test

A





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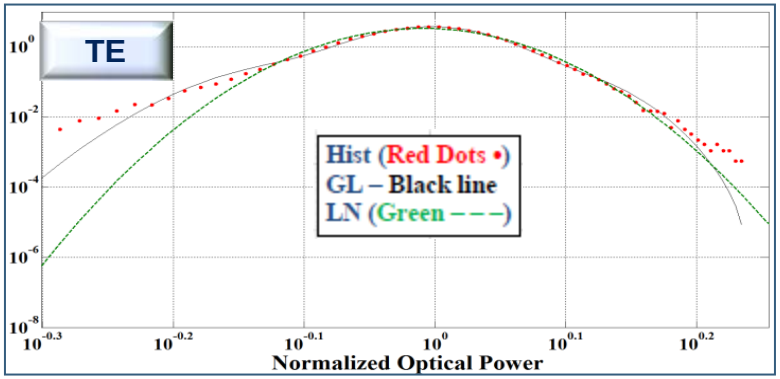

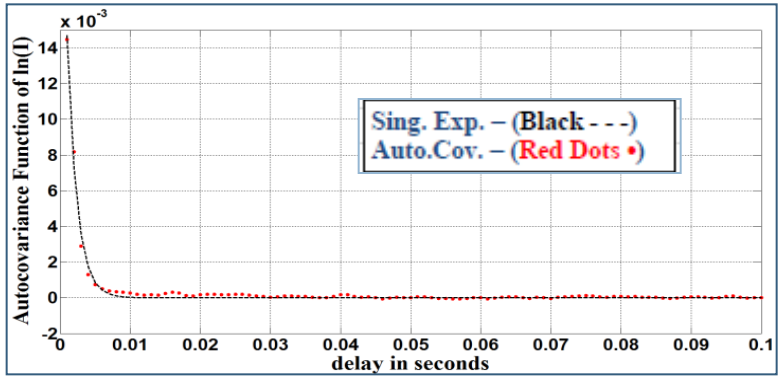

Case	N_f	D_S/ρ_0	σ_I^2	T_1	# Fades	80/100 ms	Ch. Avail.
A	0.04	0.1	0.014	0.3	3384	1/1	96.5%
B	0.04	0.1	0.014	1.5	2092	1/6	96.7%

- Similar PDFs, different Autocovariances
  - Autocovariance could relate to overall length of fades
  - D_{Det}/ρ_0 ratio might give information about overall # fades

Conclusions/Discussion

- Hot-air turbulence emulator showed a close comparison of σ_I^2, N_f, and D_S/ρ_0 with a corresponding closeness in PDFs
- The 2nd order statistics through the temporal autocovariance function and T_1 showed significant differences
  - Having a smaller T_1 could indicate shorter fade lengths
- A small D_{det}/ρ_0 ratio could indicate a higher number of fades

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